LECTURE NOTES

For Health Science Students

Biostatistics

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PREFACE

This lecture note is primarily for Health officer and Medical students who need to understand the principles of data collection, presentation, analysis and interpretation. It is also valuable to diploma students of environmental health, nursing and laboratory technology although some of the topics covered are beyond their requirements. The material could also be of paramount importance for an individual who is interested in medical or public health research.

It has been a usual practice for a health science student in Ethiopia to spend much of his/her time in search of reference materials on Biostatistics. Unfortunately, there are no textbooks which could appropriately fulfill the requirements of the Biostatistics course at the undergraduate level for Health officer and Medical students. We firmly believe that this lecture note will fill that gap.

The first three chapters cover basic concepts of Statistics focusing on the collection, presentation and summarization of data. Chapter four deals with the basic demographic methods and health service statistics giving greater emphasis to indices relating to the hospital. In chapters five and six elementary probability and sampling methods are presented with practical examples. A relatively comprehensive description of statistical inference on means and proportions is given in chapters seven and eight. The last chapter of this lecture note is about linear correlation and regression.

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Characteristics of statistical data

In order that numerical descriptions may be called statistics they must possess the following characteristics:

i) They must be in aggregates – This means that statistics are 'number of facts.' A single fact, even though numerically stated, cannot be called statistics.

ii) They must be affected to a marked extent by a multiplicity of causes.This means that statistics are aggregates of such facts only as grow out of a ' variety of circumstances'. Thus the explosion of outbreak is attributable to a number of factors, Viz., Human factors, parasite factors, mosquito and environmental factors. All these factors acting jointly determine the severity of the outbreak and it is very difficult to assess the individual contribution of any one of these factors.

iii) They must be enumerated or estimated according to a reasonable standard of accuracy – Statistics must be enumerated or estimated according to reasonable standards of accuracy. This means that if aggregates of numerical facts are to be called 'statistics' they must be reasonably accurate. This is necessary because statistical data are to serve as a basis for statistical investigations. If the basis happens to be incorrect the results are bound to be misleading.

iv) They must have been collected in a systematic manner for a predetermined purpose. Numerical data can be called statistics only if they have been compiled in a properly planned manner and for a purpose about which the enumerator had a definite idea. Facts collected in an unsystematic manner and without a complete awareness of the object, will be confusing and cannot be made the basis of valid conclusions.

v) They must be placed in relation to each other. That is, they must be comparable. Numerical facts may be placed in relation to each other either in point of time, space or condition. The phrase, 'placed in relation to each other' suggests that the facts should be comparable.

Also included in this view are the techniques for tabular and graphical presentation of data as well as the methods used to summarize a body of data with one or two meaningful figures. This aspect of organization, presentation and summarization of data are labelled as ***descriptive statistics***.

One branch of descriptive statistics of special relevance in medicine is that of vital statistics – vital events: birth, death, marriage, divorce, and the occurrence of particular disease. They are used to characterize the health status of a population. Coupled with results of

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1.3 Rationale of studying statistics

measurement? What is the magnitude and effect of laboratory and technical error? How does one interpret abnormal values?

- Statistics pervades the medical literature. As a consequence of the increasingly quantitative nature of public health and medicine and its reliance on statistical methodology, the medical literature is replete with reports in which statistical techniques are used extensively.

"It is the interpretation of data in the presence of such variability that lays at the heart of statistics."

Limitations of statistics:

It deals with only those subjects of inquiry that are capable of being quantitatively measured and numerically expressed.

1. It deals on aggregates of facts and no importance is attached to individual items–suited only if their group characteristics are desired to be studied.
2. Statistical data are only approximately and not mathematically correct.

1.4 Scales of measurement

Any aspect of an individual that is measured and take any value for different individuals or cases, like blood pressure, or records, like age, sex is called a ***variable.***

It is helpful to divide variables into different types, as different statistical methods are applicable to each. The main division is into qualitative (or categorical) or quantitative (or numerical variables).

Qualitative variable: a variable or characteristic which cannot be measured in quantitative form but can only be identified by name or categories, for instance place of birth, ethnic group, type of drug, stages of breast cancer (I, II, III, or IV), degree of pain (minimal, moderate, severe or unbearable).

Quantitative variable: A quantitative variable is one that can be measured and expressed numerically and they can be of two types (discrete or continuous). The values of a discrete variable are usually whole numbers, such as the number of episodes of diarrhoea in the first five years of life. A continuous variable is a measurement on a continuous scale. Examples include weight, height, blood pressure, age, etc.

Although the types of variables could be broadly divided into categorical (qualitative) and quantitative , it has been a common practice to see four basic types of data (scales of measurement).

Nominal data:- Data that represent categories or names. There is no implied order to the categories of nominal data. In these types of data, individuals are simply placed in the proper category or group, and the number in each category is counted. Each item must fit into exactly one category.

The simplest data consist of unordered, dichotomous, or "either - or" types of observations, i.e., either the patient lives or the patient dies, either he has some particular attribute or he does not.

eg. Nominal scale data: survival status of propanolol - treated and control patients with myocardial infarction

Status 28 days after hospital admission	Propranolol -treated patient	Control Patients
Dead	7	17
Alive	38	29
Total	45	46
Survival rate	84%	63%

Source: snow, effect of propranolol in MI ;The Lancet, 1965.

The above table presents data from a clinical trial of the drug propranolol in the treatment of myocardial infarction. There were two group of myocardial infarction. There were two group of patients with MI. One group received propranolol; the other did not and was the control. For each patient the response was dichotomous; either he

survived the first 28 days after hospital admission or he succumbed (died) sometime within this time period.

With nominal scale data the obvious and intuitive descriptive summary measure is the proportion or percentage of subjects who exhibit the attribute. Thus, we can see from the above table that 84 percent of the patients treated with propranolol survived, in contrast with only 63% of the control group.

Some other examples of nominal data:

Eye color - brown, black, etc.

Religion - Christianity, Islam, Hinduism, etc

Sex - male, female

Ordinal Data:- have order among the response classifications (categories). The spaces or intervals between the categories are not necessarily equal.

Example:

1. strongly agree
2. agree
3. no opinion
4. disagree
5. strongly disagree

In the above situation, we only know that the data are ordered.

Interval Data:- In interval data the intervals between values are the same. For example, in the Fahrenheit temperature scale, the difference between 70 degrees and 71 degrees is the same as the difference between 32 and 33 degrees. But the scale is not a RATIO Scale. 40 degrees Fahrenheit is not twice as much as 20 degrees Fahrenheit.

Ratio Data:- The data values in ratio data do have meaningful ratios, for example, age is a ratio data, some one who is 40 is twice as old as someone who is 20.

Both interval and ratio data involve measurement. Most data analysis techniques that apply to ratio data also apply to interval data. Therefore, in most practical aspects, these types of data (interval and ratio) are grouped under metric data. In some other instances, these type of data are also known as numerical discrete and numerical continuous.

Numerical discrete

Numerical discrete data occur when the observations are integers that correspond with a count of some sort. Some common examples are: the number of bacteria colonies on a plate, the number of cells within a prescribed area upon microscopic examination, the number of heart beats within a specified time interval, a mother's history of number of births (parity) and pregnancies (gravidity), the number of episodes of illness a patient experiences during some time period, etc.

Numerical continuous

The scale with the greatest degree of quantification is a numerical continuous scale. Each observation theoretically falls somewhere along a continuum. One is not restricted, in principle, to particular values such as the integers of the discrete scale. The restricting factor is the degree of accuracy of the measuring instrument most clinical measurements, such as blood pressure, serum cholesterol level, height, weight, age etc. are on a numerical continuous scale.

1.5 Exercises

Identify the type of data (nominal, ordinal, interval and ratio) represented by each of the following. Confirm your answers by giving your own examples.

1. Blood group
2. Temperature (Celsius)
3. Ethnic group
4. Job satisfaction index (1-5)
5. Number of heart attacks
6. Calendar year
7. Serum uric acid (mg/100ml)
8. Number of accidents in 3 - year period
9. Number of cases of each reportable disease reported by a health worker
10. The average weight gain of 6 1-year old dogs (with a special diet supplement) was 950grams last month.

CHAPTER TWO

Methods Of Data Collection, Organization And Presentation

2.1. Learning Objectives

At the end of this chapter, the students will be able to:

1. Identify the different methods of data organization and presentation
2. Understand the criterion for the selection of a method to organize and present data
3. Identify the different methods of data collection and criterion that we use to select a method of data collection
4. Define a questionnaire, identify the different parts of a questionnaire and indicate the procedures to prepare a questionnaire

2.2. Introduction

Before any statistical work can be done data must be collected. Depending on the type of variable and the objective of the study different data collection methods can be employed.

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such as radiographic, biochemical, X-ray machines, microscope, clinical examinations, and microbiological examinations.

Outline the guidelines for the observations prior to actual data collection.

Advantages: Gives relatively more accurate data on behavior and activities

Disadvantages: Investigators or observer's own biases, prejudice, desires, and etc. and needs more resources and skilled human power during the use of high level machines.

2. Interviews and self-administered questionnaire

Interviews and self-administered questionnaires are probably the most commonly used research data collection techniques. Therefore, designing good "***questioning tools***" forms an important and time consuming phase in the development of most research proposals.

Once the decision has been made to use these techniques, the following questions should be considered before designing our tools:

- What exactly do we want to know, according to the objectives and variables we identified earlier? Is questioning the right technique to obtain all answers, or do we need additional techniques, such as observations or analysis of records?

- Of whom will we ask questions and what techniques will we use? Do we understand the topic sufficiently to design a questionnaire, or do we need some loosely structured interviews with key informants or a focus group discussion first to orient ourselves?

- Are our informants mainly literate or illiterate? If illiterate, the use of self-administered questionnaires is not an option.
- How large is the sample that will be interviewed? Studies with many respondents often use shorter, highly structured questionnaires, whereas smaller studies allow more flexibility and may use questionnaires with a number of open-ended questions.

Once the decision has been made Interviews may be less or more structured. Unstructured interview is flexible, the content wording and order of the questions vary from interview to interview. The investigators only have idea of what they want to learn but do not decide in advance exactly what questions will be asked, or in what order.

In other situations, a more standardized technique may be used, the wording and order of the questions being decided in advance. This may take the form of a highly structured interview, in which the questions are asked orderly, or a self administered questionnaire, in which case the respondent reads the questions and fill in the answers

by himself (sometimes in the presence of an interviewer who 'stands by' to give assistance if necessary).

Standardized methods of asking questions are usually preferred in community medicine research, since they provide more assurance that the data will be reproducible. Less structured interviews may be useful in a preliminary survey, where the purpose is to obtain information to help in the subsequent planning of a study rather than factors for analysis, and in intensive studies of perceptions, attitudes, motivation and affective reactions. Unstructured interviews are characteristic of qualitative (non-quantitative) research.

The use of self-administered questionnaires is simpler and cheaper; such questionnaires can be administered to many persons simultaneously (e.g. to a class of students), and unlike interviews, can be sent by post. On the other hand, they demand a certain level of education and skill on the part of the respondents; people of a low socio-economic status are less likely to respond to a mailed questionnaire.

In interviewing using questionnaire, the investigator appoints agents known as enumerators, who go to the respondents personally with

Face-to-face and telephone interviews have many advantages. A good interviewer can stimulate and maintain the respondent's interest, and can create a rapport (understanding, concord) and

The main problems with postal questionnaire are that response rates tend to be relatively low, and that there may be under representation of less literate subjects.

3. Use of documentary sources: Clinical and other personal records, death certificates, published mortality statistics, census publications, etc. Examples include:

1. Official publications of Central Statistical Authority
2. Publication of Ministry of Health and Other Ministries
3. News Papers and Journals.
4. International Publications like Publications by WHO, World Bank,
   UNICEF
5. Records of hospitals or any Health Institutions.

During the use of data from documents, though they are less time consuming and relatively have low cost, care should be taken on the quality and completeness of the data. There could be differences in objectives between the primary author of the data and the user.

Problems in gathering data

It is important to recognize some of the main problems that may be faced when collecting data so that they can be addressed in the selection of appropriate collection methods and in the training of the staff involved.

Common problems might include:

- Language barriers
-

Primary Data

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2) The acceptability of the procedures to the subjects - the absence of inconvenience, unpleasantness, or untoward consequences.
3) The probability that the method will provide a good coverage, i.e. will supply the required information about all or almost all members of the population or sample. If many people will not know the answer to the question, the question is not an appropriate one.

The investigator's familiarity with a study procedure may be a valid consideration. It comes as no particular surprise to discover that a scientist formulates problems in a way which requires for their solution just those techniques in which he himself is specially skilled.

2.5. Types of Questions

Before examining the steps in designing a questionnaire, we need to review the types of questions used in questionnaires. Depending on how questions are asked and recorded we can distinguish two major possibilities - Open –ended questions, and closed questions.

Open-ended questions

Open-ended questions permit free responses that should be recorded in the respondent's own words. The respondent is not given any possible answers to choose from.

Such questions are useful to obtain information on:

- Facts with which the researcher is not very familiar,
- Opinions, attitudes, and suggestions of informants, or
- Sensitive issues.

For example

"Can you describe exactly what the traditional birth attendant did when your labor started?"

"What do you think are the reasons for a high drop-out rate of village health committee members?"

"What would you do if you noticed that your daughter (school girl) had a relationship with a teacher?"

Closed Questions

Closed questions offer a list of possible options or answers from which the respondents must choose. When designing closed questions one should try to:

- Offer a list of options that are exhaustive and mutually exclusive
- Keep the number of options as few as possible.

Closed questions are useful if the range of possible responses is known.

For example

“What is your marital status?

1. Single
2. Married/living together
3. Separated/divorced/widowed

“Have your every gone to the local village health worker for treatment?

1. Yes
2. No

Closed questions may also be used if one is only interested in certain aspects of an issue and does not want to waste the time of the respondent and interviewer by obtaining more information than one needs.

For example, a researcher who is only interested in the protein content of a family diet may ask:

“Did you eat any of the following foods yesterday? (Circle yes or no for each set of items)

Á	Peas, bean, lentils	Yes	No
Á			

Closed questions may be used as well to get the respondents to express their opinions by choosing rating points on a scale.

For example

"How useful would you say the activities of the Village Health Committee have been in the development of this village?"

1. Extremely useful O
2. Very useful O
3. Useful O
4. Not very useful O
5. Not useful at all O

Requirements of questions

Must have face validity – that is the question that we design should be one that give an obviously valid and relevant measurement for the variable. For example, it may be self-evident that records kept in an obstetrics ward will provide a more valid indication of birth weights than information obtained by questioning mothers.

Must be clear and unambiguous – the way in which questions are worded can '*make or break*' a questionnaire. Questions must be

clear and unambiguous. They must be phrased in language that it is believed the respondent will understand, and that all respondents will understand in the same way. To ensure clarity, each question should contain only one idea; 'double-barrelled' questions like 'Do you take your child to a doctor when he has a cold or has diarrhoea?' are difficult to answer, and the answers are difficult to interpret.

Must not be offensive – whenever possible it is wise to avoid questions that may offend the respondent, for example those that deal with intimate matters, those which may seem to expose the respondent's ignorance, and those requiring him to give a socially unacceptable answer.

The questions should be fair - They should not be phrased in a way that suggests a specific answer, and should not be loaded. Short questions are generally regarded as preferable to long ones.

Sensitive questions - It may not be possible to avoid asking 'sensitive' questions that may offend respondents, e.g. those that seem to expose the respondent's ignorance. In such situations the interviewer (questioner) should do it very carefully and wisely

2.6 Steps in Designing a Questionnaire

Designing a good questionnaire always takes several drafts. In the first draft we should concentrate on the content. In the second, we should look critically at the formulation and sequencing of the questions. Then we should scrutinize the format of the questionnaire. Finally, we should do a test-run to check whether the questionnaire gives us the information we require and whether both the respondents and we feel at ease with it. Usually the questionnaire will need some further adaptation before we can use it for actual data collection.

Step1: CONTENT

Take your objectives and variables as your starting point.

Decide what questions will be needed to measure or to define your variables and reach your objectives. When developing the questionnaire, you should reconsider the variables you have chosen, and, if necessary, add, drop or change some. You may even change some of your objectives at this stage.

Step 2: FORMULATING QUESTIONS

Formulate one or more questions that will provide the information needed for each variable.

Take care that questions are specific and precise enough that different respondents do not interpret them differently. For example, a question such as: "Where do community members usually seek treatment when they are sick?" cannot be asked in such a general way because each respondent may have something different in mind when answering the question:

- One informant may think of measles with complications and say he goes to the hospital, another of cough and say goes to the private pharmacy;

- Even if both think of the same disease, they may have different degrees of seriousness in mind and thus answer differently;
- In all cases, self-care may be overlooked.

The question, therefore, as rule has to be broken up into different parts and made so specific that all informants focus on the same thing. For example, one could:

- Concentrate on illness that has occurred in the family over the past 14 days and ask what has been done to treat if from the onset; or
- Concentrate on a number of diseases, ask whether they have occurred in the family over the past X months (chronic or serious diseases have a longer recall period than minor ailments) and what has been done to treat each of them from the onset.

Check whether each question measures one thing at a time.

For example, the question, "How large an interval would you and your husband prefer between two successive births?" would better be divided into two questions because husband and wife may have different opinions on the preferred interval.

Avoid leading questions.

A question is leading if it suggests a certain answer. For example, the question, ''Do you agree that the district health team should visit each health center monthly?'' hardly leaves room for “no” or for other options. Better would be: “Do you thing that district health teams should visit each health center? If yes, how often?”

Sometimes, a question is leading because it presupposes a certain condition. For example: “What action did you take when your child had diarrhoea the last time?” presupposes the child has had diarrhoea. A better set of questions would be: “Has your child had diarrhoea? If yes, when was the last time?” “Did you do anything to treat it? If yes, what?”

Step 3: SEQUENCING OF QUESTIONS

Design your interview schedule or questionnaire to be “consumer friendly.”

- The sequence of questions must be logical for the respondent and allow as much as possible for a “natural” discussion, even in more structured interviews.
- At the beginning of the interview, keep questions concerning “background variables” (e.g., age, religion, education, marital status, or occupation) to a minimum. If possible, pose most or all of these questions later in the interview. (Respondents

may be reluctant to provide “personal” information early in an interview)

- Start with an interesting but non-controversial question (preferably open) that is directly related to the subject of the study. This type of beginning should help to raise the informants’ interest and lessen suspicions concerning the purpose of the interview (e.g., that it will be used to provide information to use in levying taxes).
- Pose more sensitive questions as late as possible in the interview (e.g., questions pertaining to income, sexual behavior, or diseases with stigma attached to them, etc.
- Use simple everyday language.

Make the questionnaire as short as possible. Conduct the interview in two parts if the nature of the topic requires a long questionnaire (more than 1 hour).

Step 4: FORMATTING THE QUESTIONNAIRE

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A study in which 400 persons were asked how many full-length movies they had seen on television during the preceding week. The following gives the distribution of the data collected.

Number of movies	Number of persons	Relative frequency (%)
0	72	18.0
1	106	26.5
2	153	38.3
3	40	10.0
4	18	4.5
5	7	1.8
6	3	0.8
7	0	0.0
8	1	0.3
Total	400	100.0

In the above distribution Number of movies represents the variable under consideration, Number of persons represents the frequency, and the whole distribution is called frequency distribution particularly simple frequency distribution.

A categorical distribution – non-numerical information can also be represented in a frequency distribution. Seniors of a high school were interviewed on their plan after completing high school. The following data give plans of 548 seniors of a high school.

SENIORS' PLAN	NUMBER OF SENIORS
Plan to attend college	240
May attend college	146
Plan to or may attend a vocational school	57
Will not attend any school	105
Total	548

Consider the problem of a social scientist who wants to study the age of persons arrested in a country. In connection with large sets of data, a good overall picture and sufficient information can often be conveyed by grouping the data into a number of class intervals as shown below.

Age (years)	**Number of persons**
Under 18	1,748
18 – 24	3,325
25 – 34	3,149
35 – 44	1,323
45 – 54	512
55 and over	335
Total	10,392

This kind of frequency distribution is called grouped frequency distribution.

Frequency distributions present data in a relatively compact form, gives a good overall picture, and contain information that is adequate for many purposes, but there are usually some things which can be determined only from the original data. For instance, the above grouped frequency distribution cannot tell how many of the arrested persons are 19 years old, or how many are over 62.

The construction of grouped frequency distribution consists essentially of four steps:
(1) Choosing the classes, (2) sorting (or tallying) of the data into these classes, (3) counting the number of items in each class, and (4) displaying the results in the forma of a chart or table

Choosing suitable classification involves choosing the number of classes and the range of values each class should cover, namely, from where to where each class should go. Both of these choices are arbitrary to some extent, but they depend on the nature of the data

A guide on the determination of the number of classes (k) can be the Sturge's Formula, given by:

K = 1 + 3.322×log(n), where n is the number of observations

And the length or width of the class interval (w) can be calculated by:

W = (Maximum value – Minimum value)/K = Range/K

2) We always make sure that each item (measurement or observation) goes into one and only one class, i.e. classes should be mutually exclusive. To this end we must make sure that the smallest and largest values fall within the classification, that none of the values can fall into possible gaps between successive classes, and that the classes do not overlap, namely, that successive classes have no values in common.

Note that the Sturges rule should not be regarded as final, but should be considered as a guide only. The number of classes specified by the rule should be increased or decreased for convenient or clear presentation.

3) **Determination of class limits**: (i) Class limits should be definite and clearly stated. In other words, open-end classes should be avoided since they make it difficult, or even impossible, to calculate certain further descriptions that may be of interest. These are classes like less then 10, greater than 65, and so on. (ii) The starting point, i.e., the

lower limit of the first class be determined in such a manner that frequency of each class get concentrated near the middle of the class interval. This is necessary because in the interpretation of a frequency table and in subsequent calculation based up on it, the mid-point of each class is taken to represent the value of all items included in the frequency of that class.

It is important to watch whether they are given to the nearest inch or to the nearest tenth of an inch, whether they are given to the nearest ounce or to the nearest hundredth of an ounce, and so forth. For instance, to group the weights of certain animals, we could use the first of the following three classifications if the weights are given to the nearest kilogram, the second if the weights are given to the nearest tenth of a kilogram, and the third if the weights are given to the nearest hundredth of a kilogram:

Weight (kg)	**Weight (kg)**	**Weight (kg)**
10 – 14	10.0 – 14.9	10.00 – 14.99
15 – 19	15.0 – 19.9	15.00 – 19.99
20 – 24	20.0 – 24.9	20.00 – 24.99
25 – 29	25.0 – 29.9	25.00 – 29.99
30 – 34	30.0 – 34.9	30.00 – 34.99

Example: Construct a grouped frequency distribution of the following data on the amount of time (in hours) that 80 college students devoted to leisure activities during a typical school week:

23	24	18	14	20	24	24	26	23	21
16	15	19	20	22	14	13	20	19	27
29	22	38	28	34	32	23	19	21	31
16	28	19	18	12	27	15	21	25	16
30	17	22	29	29	18	25	20	16	11
17	12	15	24	25	21	22	17	18	15
21	20	23	18	17	15	16	26	23	22
11	16	18	20	23	19	17	15	20	10

Using the above formula, K = 1 + 3.322 $\times$ log (80) = 7.32 $\approx$ 7 classes

Maximum value = 38 and Minimum value = 10 Č Range = 38 – 10 = 28 and W = 28/7 = 4

Using width of 5, we can construct grouped frequency distribution for the above data as:

Time spent (hours)	Tally	Frequency	Cumulative freq
10 – 14	/// ///	8	8
15 – 19	/// /// /// /// /// ///	28	36
20 – 24	/// /// /// /// /// //	27	63
25 – 29	/// /// //	12	75
30 – 34	////	4	79
35 – 39	/	1	80

The smallest and largest values that can go into any class are referred to as its class limits; they can be either lower or upper class limits.

For our data of patients, for example

n = 50 then $k = 1 + 3.322(\log_{10}50) = 6.64 = 7$ and $w = R / k = (89 - 1)/7 = 12.57 = 13$

Cumulative and Relative Frequencies: When frequencies of two or more classes are added up, such total frequencies are called Cumulative Frequencies. This frequencies help as to find the total number of items whose values are less than or greater than some value. On the other hand, relative frequencies express the frequency of each value or class as a percentage to the total frequency.

Note. In the construction of cumulative frequency distribution, if we start the cumulation from the lowest size of the variable to the highest size, the resulting frequency distribution is called `**Less than cumulative frequency distribution**' and if the cumulation is from the highest to the lowest value the resulting frequency distribution is called `**more than cumulative frequency distribution.'** The most common cumulative frequency is the less than cumulative frequency.

Mid-Point of a class interval and the determination of Class Boundaries

Mid-point or class mark (Xc) of an interval is the value of the interval which lies mid-way between the lower true limit (LTL) and the upper true limit (UTL) of a class. It is calculated as:

$$X_c = \frac{\text{Upper Class Limit} + \text{Lower Class Limit}}{2}$$

True limits (or class boundaries) are those limits, which are determined mathematically to make an interval of a continuous variable continuous in both directions, and no gap exists between classes. The true limits are what the tabulated limits would correspond with if one could measure exactly.

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Example: Frequency distribution of weights (in Ounces) of Malignant Tumors Removed from the Abdomen of 57 subjects

Weight	Class boundaries	Xc	Freq.	Cum. freq.	Relative freq (%)
10 - 19	9.5 - 19.5	14.5	5	5	0.0877
20 - 29	19.5 - 29.5	24.5	19	24	0.3333
30 - 39	29.5 - 39.5	34.5	10	34	0.1754
40 - 49	39.5 - 49.5	44.5	13	47	0.2281
50 - 59	49.5 - 59.5	54.5	4	51	0.0702
60 - 69	59.5 - 69.5	64.5	4	55	0.0702
70 - 79	69.5 - 79.5	74.5	2	57	0.0352
Total			57		1.0000

For example, the width of the above distribution is (let's take the fourth class) w = 49.5 - 39.5 = 10.

2.7.2 Statistical Tables

A statistical table is an orderly and systematic presentation of numerical data in rows and columns. Rows (stubs) are horizontal and columns (captions) are vertical arrangements. The use of tables for organizing data involves grouping the data into mutually exclusive categories of the variables and counting the number of occurrences (frequency) to each category.

These mutually exclusive categories, for qualitative variables, are naturally occurring groupings. For example, Sex (Male, Female), Marital status (single, Married, divorced, widowed, etc.), Blood group (A, B, AB, O), Method of Delivery (Normal, forceps, Cesarean section, etc.), etc. are some qualitative variables with exclusive categories.

In the case of large size quantitative variables like weight, height, etc. measurements, the groups are formed by amalgamating continuous values into classes of intervals. There are, however, variables which have frequently used standard classes. One of such variables, which have wider applications in demographic surveys, is age. The age distribution of a population is described based on the following intervals:

< 1	20-24	45-49
1-4	25-29	50-54
5-9	30-34	55-59
10-14	35-39	60-64
15-19	40-44	65+

Based on the purpose for which the table is designed and the complexity of the relationship, a table could be either of simple frequency table or cross tabulation.

The simple frequency table is used when the individual observations involve only to a single variable whereas the cross tabulation is used to obtain the frequency distribution of one variable by the subset of another variable. In addition to the frequency counts, the relative frequency is used to clearly depict the distributional pattern of data. It shows the percentages of a given frequency count. For simple frequency distributions, (like Table 1) the denominators for the percentages are the sum of all observed frequencies, i.e. 210.

On the other hand, in cross tabulated frequency distributions where there are row and column totals, the decision for the denominator is based on the variable of interest to be compared over the subset of

distribution of one variable by the subset of another variable. In addition to the frequency counts, the relative frequency is used to clearly depict the distributional pattern of data. It shows the percentages of a given frequency count.

Table 1: Overall immunization status of children in Adami Tullu Woreda, Feb. 1995

Immunization status	Number	Percent
Not immunized	75	35.7
Partially immunized	57	27.1
Fully immunized	78	37.2
Total	210	100.0

Source: Fikru T et al. EPI Coverage in Adami Tulu. Eth J Health Dev 1997;11(2): 109-113

B. Two-way table: This table shows two characteristics and is formed when either the caption or the stub is divided into two or more parts.

In cross tabulated frequency distributions where there are row and column totals, the decision for the denominator is based on the variable of interest to be compared over the subset of the other variable. For example, in Table 2 the interest is to compare the immunization status of mothers in different marital status group.

under each marital status group will be the total numbers of mothers in each marital status category, i.e. row total.

Table 2: TT immunization by marital status of the women of childbearing age, Assendabo town, Jimma Zone, 1996

Marital Status	Immunization Status				Total
	Immunized		Non Immunized		
	No.	%	No.	%	
Single	58	24.7	177	75.3	235
Married	156	34.7	294	65.3	450
Divorced	10	35.7	18	64.3	28
Widowed	7	50.0	7	50.0	14
Total	231	31.8	496	68.2	727

sample a cross-tabulation was constructed which included the sex and the residence (rural urban) of the doctors and nurses interviewed.

Table 3: Distribution of Health Professional by Sex and Residence

		Residence		
Profession/Sex		Urban	Rural	Total
Doctors	Male	8 (10.0)	35 (21.0)	43 (17.7)
	Female	2 (3.0)	16 (10.0)	18 (7.4)
Nurses	Male	46 (58.0)	36 (22.0)	82 (33.7)
	Female	23 (29.0)	77 (47.0)	100 (41.2)
Total		79 (100.0)	164 (100.0)	243 (100.0)

2.7.3. Diagrammatic Representation of Data

Appropriately drawn graph allows readers to obtain rapidly an overall grasp of the data presented. The relationship between numbers of various magnitudes can usually be seen more quickly and easily from a graph than from a table.

Figures are not always interesting, and as their size and number increase they become confusing and uninteresting to such an extent that no one (unless he is specifically interested) would care to study them. Their study is a greater strain upon the mind without, in mo(ou) p.4.8 abs TD91.0109 T

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2. Diagrammatic representation is not an alternative to tabulation. It only strengthens the textual exposition of a subject, and cannot serve as a complete substitute for statistical data.
3. It can give only an approximate idea and as such where greater accuracy is needed diagrams will not be suitable.
4. They fail to bring to light small differences

Construction of graphs

The choice of the particular form among the different possibilities will depend on personal choices and/or the type of the data.

- Bar charts and pie chart are commonly used for qualitative or quantitative discrete data.
- Histograms, frequency polygons are used for quantitative continuous data.

There are, however, general rules that are commonly accepted about construction of graphs.

1. Every graph should be self-explanatory and as simple as possible.
2. Titles are usually placed below the graph and it should again question what ? Where? When? How classified?
3. Legends or keys should be used to differentiate variables if more than one is shown.

4. The axes label should be placed to read from the left side and from the bottom.

5. The units in to which the scale is divided should be clearly indicated.

6. The numerical scale representing frequency must start at zero or

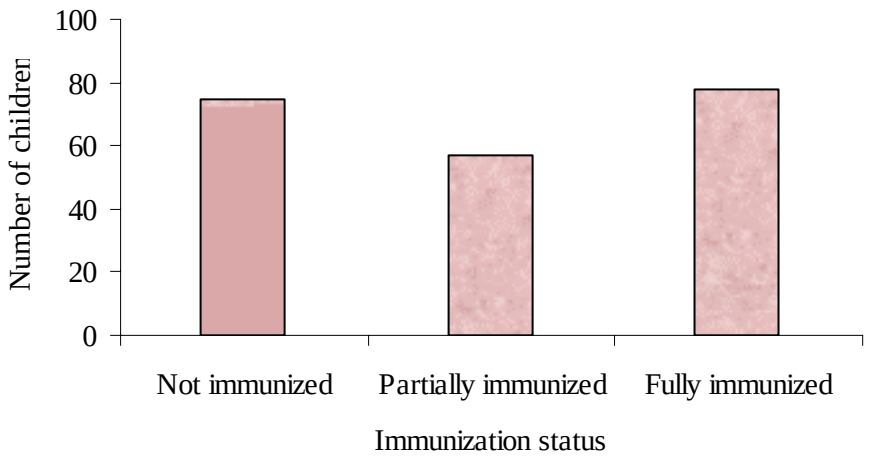

Fig. 1. Immunization status of Children in Adami Tulu Woreda, Feb. 1995

B. ***Multiple bar chart:*** In this type of chart the component figures are shown as separate bars adjoining each other. The height of each bar represents the actual value of the component figure. It depicts distributional pattern of more than one variable

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Example of actual component bar diagram: The above data can also be presented as below.

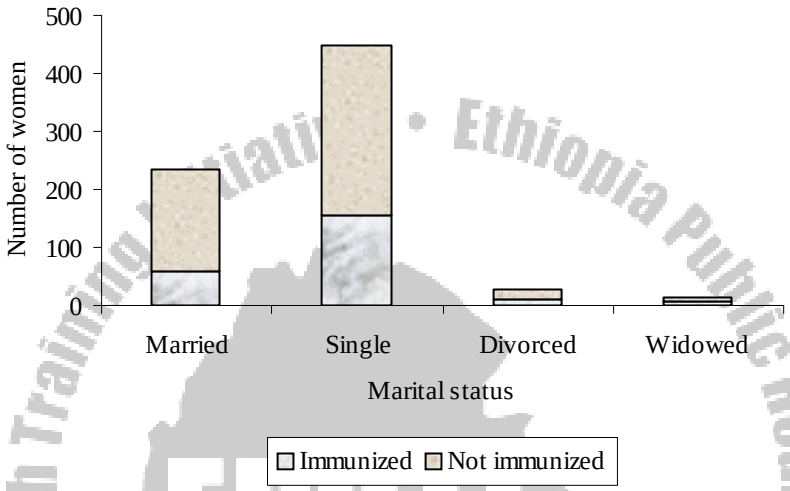


Fig. 3 TT Immunization status by marital status of women 15-49 years, Asendabo town, 1996

ii) Percentage Component Bar Diagram: Where the individual component lengths represent the percentage each component forms the over all total. Note that a series of such bars will all be the same total height, i.e., 100 percent.

Example of percentage component bar diagram:

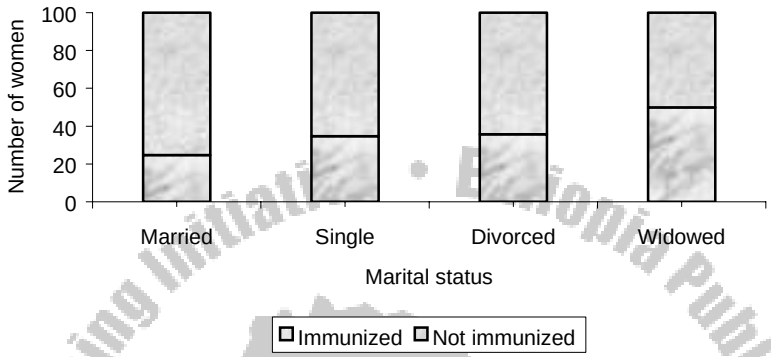
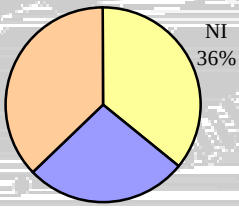


Fig. 4 TT Immunization status by marital status of women 15-49 years, Asendabo town, 1996

2) Pie-chart (qualitative or quantitative discrete data): it is a circle divided into sectors so that the areas of the sectors are proportional to the frequencies.



3. Histograms (quantitative continuous data)

A histogram is the graph of the frequency distribution of continuous measurement variables. It is constructed on the basis of the following principles:

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Fig 6: Histogram for amount of time college students devoted to leisure activities

2. FREQUENCY POLYGON:

If we join the midpoints of the tops of the adjacent rectangles of the histogram with line segments a frequency polygon is obtained. When the polygon is continued to the X-axis just out side the range of the lengths the total area under the polygon will be equal to the total area under the histogram.

Note that it is not essential to draw histogram in order to obtain frequency polygon. It can be drawn with out erecting rectangles of histogram as follows:

1) The scale should be marked in the numerical values of the mid-points of intervals.
2) Erect ordinates on the midpoints of the interval - the length or altitude of an ordinate representing the frequency of the class on whose mid-point it is erected.
3) Join the tops of the ordinates and extend the connecting lines to the scale of sizes.

Example: Consider the above data on time spend on leisure activities

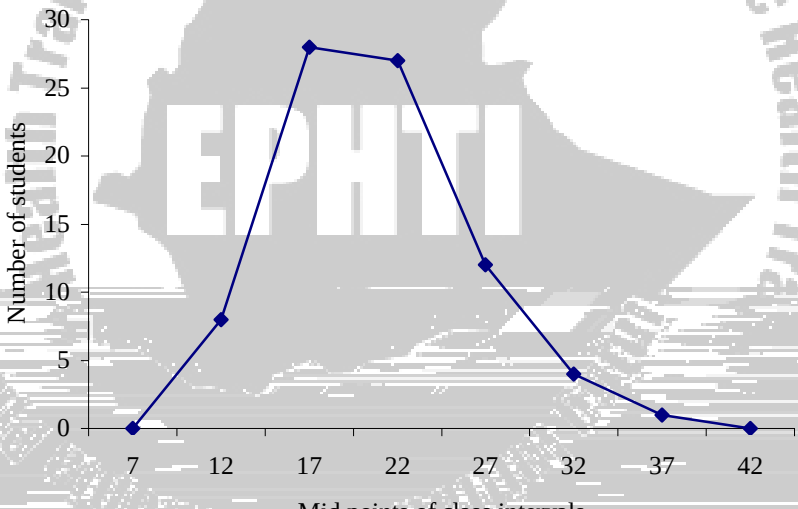


Fig 7: Frequency polygon curve on time spent for leisure activities by students

3. **OGIVE OR CUMULATIVE FREQUENCY CURVE:** When the **cumulative frequencies** of a distribution are graphed the resulting curve is called **Ogive Curve**.

To construct an Ogive curve:

i) Compute the cumulative frequency of the distribution.

ii) Prepare a graph with the cumulative frequency on the vertical axis and the true upper class limits (class boundaries) of the interval scaled along the X-axis (horizontal axis). The true lower limit of the lowest class interval with lowest scores is included in the X-axis scale; this is also the true upper limit of the next lower interval having a cumulative frequency of 0.

Example: Consider the above data on time spend on leisure activities

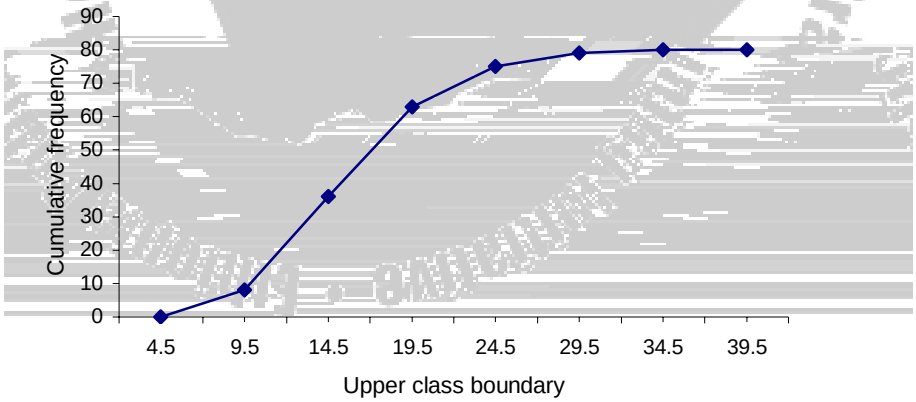
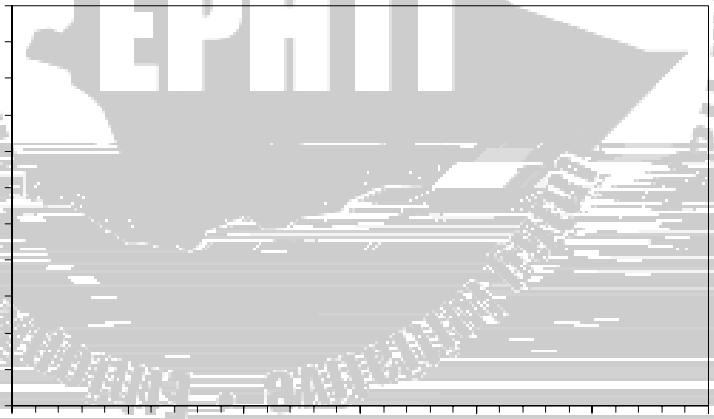


Fig 8: Cumulative frequency curve for amount of time college students devoted to leisure activities

4. The line diagram

The line graph is especially useful for the study of some variables according to the passage of time. The time, in weeks, months or years is marked along the horizontal axis; and the value of the quantity that is being studied is marked on the vertical axis. The distance of each plotted point above the base-line indicates its numerical value. The line graph is suitable for depicting a consecutive trend of a series over a long period.

Example: Malaria parasite rates as obtained from malaria seasonal blood survey results, Ethiopia (1967-79 E.C)



CHAPTER THREE

Summarizing Data

3.1. Learning objectives

At the end of this chapter, the student will be able to:

1. Identify the different methods of data summarization
2. Compute appropriate summary values for a set of data
3. Appreciate the properties and limitations of summary values

3.2. Introduction

The first step in looking at data is to describe the data at hand in some concise way. In smaller studies this step can be accomplished by listing each data point. In general, however, this procedure is tedious or impossible and, even if it were possible would not give an over-all picture of what the data look like.

The basic problem of statistics can be stated as follows: Consider a sample of data X_1, ……. X_n, where X_1 corresponds to the first sample point and X_n corresponds to the nth sample point. Presuming that the sample is drawn from some population P, what inferences or conclusion can be made about P from the sample?

Before this question can be answered, the data must be summarized as succinctly (concisely, briefly) as possible, since the number of sample points is frequently large and it is easy to lose track of the overall picture by looking at all the data at once. One type of measure useful for summarizing data defines the center, or middle, of the sample. This type of measure is a measure of central tendency (location).

Before attempting the measures of central tendency and dispersion, let's see some of the notations that are used frequently.

Notations: $\sum$ is read as Sigma (the Greek Capital letter for S) means the sum of

Suppose n values of a variable are denoted as $x_1, x_2, x_3 \ldots, x_n$ then $\sum x_i = x_1, +x_2, + x_3 + \ldots x_n$ where the subscript i range from 1 up to n

Example: Let $x_1=2$, $x_2 = 5$, $x_3=1$, $x_4 =4$, $x_5=10$, $x_6= -5$, $x_7 = 8$

Since there are 7 observations, i range from 1 up to 7

i) $\sum x_i = 2+5+1+4+10-5+8 = 25$

ii) $(\sum x_i)^2 = (25)^2 = 625$

iii) $\sum x_i^2 = 4 + 25 + 1 + 16 + 100 + 25 + 64 = 235$

Rules for working with summation

1) $\sum(x$

of sample. Nevertheless, the arithmetic mean is by far the most widely used measure of central location.

2. Median

An alternative measure of centron. wi3 location.

the middle 2 values must be averaged. Thus, for sample of size 8, the fourth and the fifth largest points would be averaged to obtain the median, since neither is the central point.

Example: Compute the sample median for the birth weight data

Solution: First arrange the sample in ascending order
2069, 2581, 2759, 2834, 2838, 2841, 3031, 3101, 3200, 3245, 3248, 3260, 3265, 3314, 3323, 3484, 3541, 3609, 3649, 4146

Since n=20 is even,
Median = average of the 10^{th} and 11^{th} largest observation = (3245 + 3248)/2 = 3246.5 g

Example: Consider the following data, which consists of white blood counts taken on admission of all patients entering a small hospital on a given day. Compute the median white-blood count ($\times 10^3$). 7, 35,5,9,8,3,10,12,8

Solution: First, order the sample as follows. 3,5,7,8,8,9,10,12,35. Since n is odd, the sample median is given by the 5^{th}, $((9+1)/2)^{th}$, largest point, which is equal to 8.
The principal strength of the sample median is that it is **insensitive to very large or very small values**.

In particular, if the second patient in the above data had a white blood count of 65,000 rather than 35,000, the sample median would remain unchanged, since the fifth largest value is still 8,000. Conversely the arithmetic mean would increase dramatically from 10,778 in the original sample to 14,111 in the new sample.

The principal weakness of the sample median is that it is determined mainly by the middle points in a sample and is less sensitive to the actual numerical values of the remaining data points.

3. Mode

It is the value of the observation that occurs with the greatest frequency. A particular disadvantage is that, with a small number of observations, there may be no mode. In addition, sometimes, there may be more than one mode such as when dealing with a bimodal (two-peaks) distribution. It is even less amenable (responsive) to mathematical treatment than the median. The mode is not often used in biological or medical data.

Find the modal values for the following data

a) 22, 66, 69, 70, 73. (no modal value)

b) 1.8, 3.0, 3.3, 2.8, 2.9, 3.6, 3.0, 1.9, 3.2, 3.5 (modal value = 3.0 kg)

Skewness: If extremely low or extremely high observations are present in a distribution, then the mean tends to shift towards those scores. Based on the type of skewness, distributions can be:

a) **Negatively skewed distribution**: occurs when majority of scores are at the right end of the curve and a few small scores are scattered at the left end.

b) **Positively skewed distribution:** Occurs when the majority of scores are at the left end of the curve and a few extreme large scores are scattered at the right end.

c) **Symmetrical distribution**: It is neither positively nor negatively skewed. A curve is symmetrical if one half of the curve is the mirror image of the other half.

In unimodal (one-peak) symmetrical distributions, the mean, median and mode are identical. On the other hand, in unimodal skewed distributions, it is important to remember that the mean, median and mode occur in alphabetical order when the longer tail is at the left of the distribution or in reverse alphabetical order when the longer tail is at the right of the distribution.

4. Geometric mean: It is obtained by taking the n^{th} root of the product

The geometric mean is preferable to the arithmetic mean if the series of observations contains one or more unusually large values. The above method of calculating geometric mean is satisfactory only if there are a small number of items. But if n is a large number, the problem of computing the n^{th} root of the product of these values by simple arithmetic is a tedious work. To facilitate the computation of geometric mean we make use of logarithms. The above formula when reduced to its logarithmic form will be:

$GM = \sqrt[n]{(x1)(x2)\ldots.(x_n)} = \{ (x1)(x2)\ldots (x_n) \}^{1/n}$

$\text{Log GM} = \log \{(x_1)(x_2)\ldots(x_n)\}^{1/n}$

$= 1/n \log \{(x_1)(x_2)\ldots(x_n)\}$

$=1/n \{\log(x_1) + \log(x_2) + \ldots \log(x_n)\}$

$= \sum(\log xi)/n$

The logarithm of the geometric mean is equal to the arithmetic mean of the logarithms of individual values. The actual process involves obtaining logarithm of each value, adding them and dividing the sum by the number of observations. The quotient so obtained is then looked up in the tables of anti-logarithms which will give us the geometric mean.

Examle: The geometric mean may be calculated for the following parasite counts per 100 fields of thick films.

7	8	3	14	2	1	440	15	52	6	2	1	1	25
12	6	9	2	1	6	7	3	4	70	20	200	2	50
21	15	10	120	8	4	70	3	1	103	20	90	1	237

$GM = \sqrt[42]{7x8x3x\ldots x1x237}$

log Gm = 1/42 (log 7+log8+log3+..+log 237)

= 1/42 (.8451+.9031+.4771 +…2.3747)

= 1/42 (41.9985)

= 0.9999 ≈ 1.0000

The anti-log of 0.9999 is 9.9992 ≈

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iv) Disadvantages

1) It may be greatly affected by extreme items and its usefulness as a “Summary of the whole” may be

C. Mode

i) Characteristics

1) It is an average of position
2) It is not affected by extreme values
3) It is the most typical value of the distribution

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3. For any series of items it is always smaller than the arithmetic mean.
4. It exists ordinarily only for positive values.

ii) Advantages:-

1) since it is less affected by extremes it is a more preferable average than the arithmetic mean
2) It is capable of algebraic treatment
3) It based on all values given in the distribution.

iv) Disadvantages:-

1) Its computation is relatively difficult.
2) It cannot be determined if there is any negative value in the distribution, or where one of the items has a zero value.

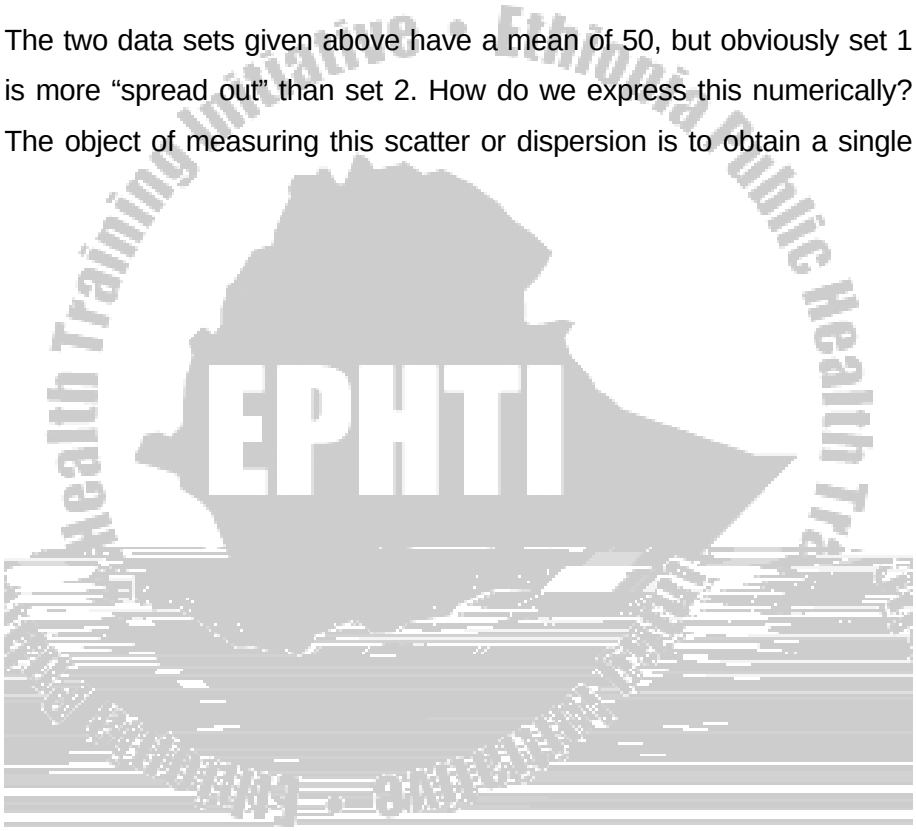
3.4. Measures of Variation

In the preceding sections several measures which are used to describe the central tendency of a distribution were considered. While the mean, median, etc. give useful information about the center of the data, we also need to know how "spread out" the numbers are abut the center.

Consider the following data sets:

								Mean
Set 1:	60	40	30	50	60	40	70	50
Set 2:	50	49	49	51	48	50	53	50

The two data sets given above have a mean of 50, but obviously set 1 is more “spread out” than set 2. How do we express this numerically? The object of measuring this scatter or dispersion is to obtain a single



Where , x_{max} = highest (maximum) value in the given distribution.

Xmin = lowest (minimum) value in the given distribution.

In our example given above (the two data sets)

* The range of data in set 1 is 70-30 =40
* The range of data in set 2 is 53-48 =5

1. Since it is based upon two extreme cases in the entire distribution, the range may be considerably changed if either of the extreme cases happens to drop out, while the removal of any other case would not affect it at all.

2. It wastes information for it takes no account of the entire data.

3. The extremes values may be unreliable; that is, they are the most likely to be faulty

4. Not suitable with regard to the mathematical treatment required in

such that p percent of the sample points are less than or equal to Vp. The median, being the 50^{th} percentile, is a special case of a quantile. As was the case for the median, a different definition is needed for the p^{th} percentile, depending on whether np/100 is an integer or not.

Definition: The p^{th} percentile is defined by

(1) The $(k+1)^{th}$ largest sample point if np/100 is not an integer (where k is the largest integer less than np/100)

(2) The average of the $(np/100)^{th}$ and $(np/100 + 1)^{th}$ largest observation is np/100 is an integer.

The spread of a distribution can be characterized by specifying several percentiles. For example, the 10^{th} and 90^{th} percentiles are often used to characterize spread. Percentages have the advantage over the range of being less sensitive to outliers and of not being much affected by the sample size (n).

Example: Compute the 10^{th} and 90^{th} percentile for the birth weight data.

Solution: Since $20\times0.1=2$ and $20\times0.9=18$ are integers, the 10^{th} and 90^{th} percentiles are defined by

10^{th} percentile = the average of the 2^{nd} and 3^{rd} largest values = (2581+2759)/2 = 2670 g

Biostatistics

18 21 23 24 24 32 42 59

1st quartile = The $\{(n+1)/4\}^{th}$ observation = $(2.25)^{th}$ observation

= 21 + (23-21)x .25 = 21.5

3rd quartile = $\{3/4\ (n+1)\}^{th}$ observation = $(6.75)^{th}$ observation

= 32 + (42-32)x .75 = 39.5

Hence, IQR = 39.5 - 21.5 = 18

The interquartile range is a preferable measure to the range. Because it is less prone to distortion by a single large or small value. That is, outliers in the data do not affect the inerquartile range. Also, it can be computed when the distribution has open-end classes.

3. Standard Deviation and Variance

Definition: The sample and population standard deviations denoted by S and σ (by convention) respectively are defined as follows:

$$S = \sqrt{\qquad\qquad}$$

$\sigma = \sqrt{\frac{\sum (X_i - \mu)^2}{N}}$ = population standard deviation

This measure of variation is universally used to show the scatter of the individual measurements around the mean of all the measurements in a given distribution.

Note that the sum of the deviations of the individual observations of a sample about the sample mean is always 0.

The square of the standard deviation is called the variance. The variance is a very useful measure of variability because it uses the information provided by every observation in the sample and also it is very easy to handle mathematically. Its main disadvantage is that the units of variance are the square of the units of the original observations. Thus if the original observations were, for example, heights in cm then the units of variance of the heights are cm^2. The easiest way around this difficulty is to use the square root of the variance (i.e., standard deviation) as a measure of variability.

Computational formulas for the sample variance or SD

$S^2 = \frac{\sum_{i=1}^{n} X_i^2 - n\bar{X}^2}{n-1}$ and $S^2 = \frac{n\sum_{i=1}^{n} X_i^2 - (\sum_{i=1}^{n} X_i)^2}{n-1}$

$$S = \sqrt{\frac{\sum_{i=1}^{n} X_i - n\overline{X}^2}{n-1}} \quad \text{and} \quad S = \sqrt{\frac{n\sum_{i=1}^{n} X_i^2 - (\sum_{i=1}^{n} X_i)^2}{n(n-1)}}$$

Example: Areas of sprayable surfaces with DDT from a sample of 15 houses are as follows (m^2) :

101,105,110,114,115,124,125, 125, 130,133,135,136,137,140,145

Find the variance and standard deviation of the above distribution.

The mean of the sample is 125 m^2.

Variance (sample) = $s^2 = \sum(xi - x)^2/n-1$

$= \{(101-125)^2 + (105-125)^2 + \ldots.(145-125)^2\} / (15-1)$

$= 2502/14$

$= 178.71$ (square metres)^2

Hence, the standard deviation = $\sqrt{178.71} = 13.37$ m^2.

Some important properties of the arithmetic mean and standard deviation

Consider a sample $X_1, \ldots.., X_n$, which will be referred to as the original sample. To create a translated sample X_1+c, add a constant C to

each data point. Let $y_i = x_i+c$, i = 1, …., n. Suppose we want to compute the arithmetic mean of the translated sample, we can show that the following relationship holds:

1. If $Y_i = X_i + c$, i = 1, …., n

Then $\overline{Y} = \overline{X} + c$ and Sy = Sx

Therefore, to find the arithmetic mean of the Y's, compute the arithmetic mean of the X's and add the constant c but the standard deviation of Y will be the same as the standard deviation of X.

This principle is useful because it is sometimes convenient to change the "origin" of the sample data, that is, compute the arithmetic mean after the translation and transform back to the original data.

What happens to the arithmetic mean if the units or scales being worked with are changed? A re-scaled sample can be created:

2. If $Y_i = cx_i$, i=1, ……., n

Then $\overline{Y} = c\overline{X}$ and Sy = cSx

Therefore, to find the arithmetic mean and standard deviation of the Y's compute the arithmetic mean and standard deviation of the X's and multiply it by the constant c.

Example: Express the mean and standard deviation of birth weight for the above data in ounces rather than grams.

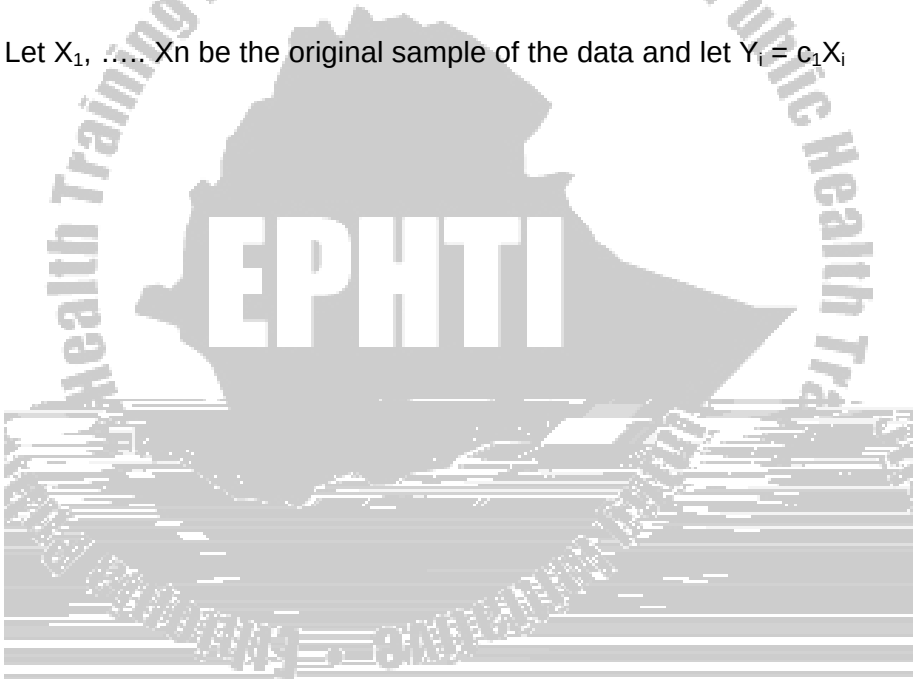
We know that 1 oz = 28.35 gm and that $\overline{X}$ = 3166.9 gm and S_x = 445.3 gm. Thus, if the data were expressed in terms of ounces:

$c = \frac{1}{28.35}$ and $\overline{Y} = \frac{1}{28.35}(3166.9)$ = 111.71 oz and

S_y = 1/28.35(445.3) = 15.7 oz

Sometimes we want to change both the origin and the scale of the data at the same time. To do this,

Let X_1, ….. Xn be the original sample of the data and let $Y_i = c_1X_i$



Then the arithmetic mean and standard deviation in ^{o}F would be

$\overline{Y} = \frac{9}{5}(11.75) + 32 = 53.15°F$ and Sy = 9/5(1.8) = 3.24 ^{0}F

Weighted Mean of Sample Means and Pooled Standard Deviation

When averaging quantities, it is often necessary to account for the fact that not all of them are equally important in the phenomenon being described. In order to give quantities being averaged there proper degree of importance, it is necessary to assign them relative importance called ***weights***, and then calculate a weighted mean. In general, the weighted mean $\overline{X}_w$ of a set of numbers X_1, X_2, … and X_n, whose relative importance is expressed numerically by a corresponding set of numbers w_1, w_2, … and w_n, is given by

$$\overline{X}_w = \frac{w_1X_1 + w_2X_2 + ... + w_nX_n}{w_1 + w_2 + ... + w_n} = \frac{\sum w \times X}{\sum w}$$

Example: In a given drug shop four different drugs were sold for unit price of 0.60, 0.85, 0.95 and 0.50 birr and the total number of drugs sold were 10, 10, 5 and 20 respectively. What is the average price of the four drugs in this drug shop?

Solution: for this example we have to use weighted mean using number of drugs sold as the respective weights for each drug's price. Therefore, the average price will be:

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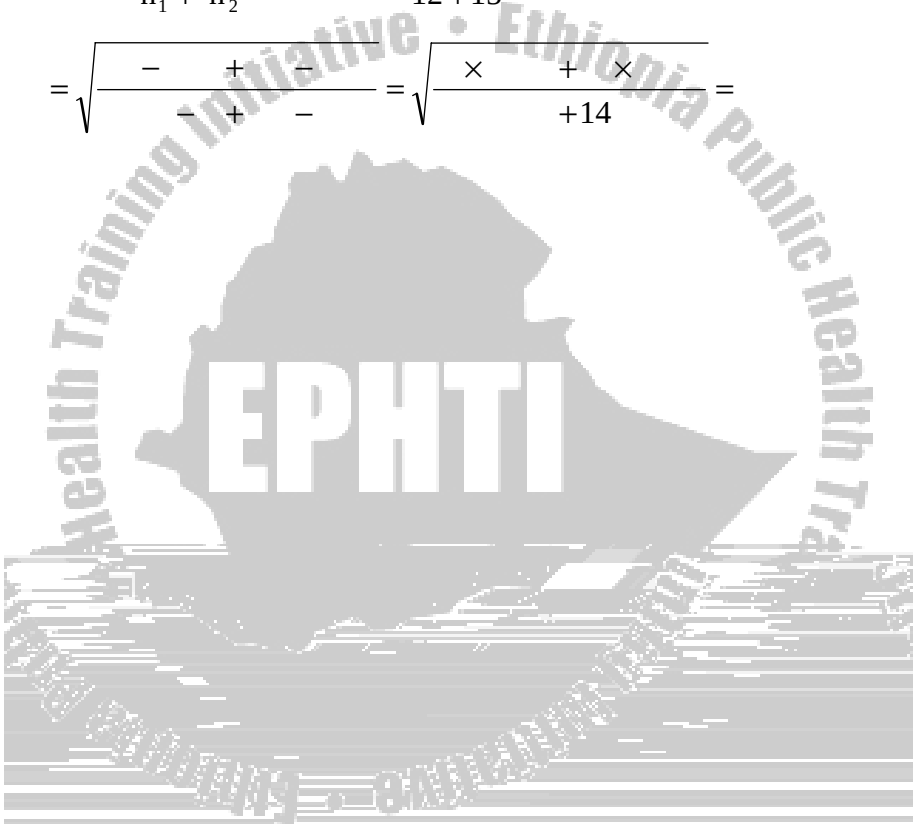
The mean of the 27 men is given by the weighted mean of the two groups.

$$\overline{X}_W = \frac{n_1\overline{X}_1 + n_2\overline{X}_2}{n_1 + n_2} = \frac{12(129.4) + 15(133.6)}{12 + 15} = 131.73\,\text{mm Hg}$$

$$= \sqrt{\frac{- \quad + \quad -}{- \quad + \quad -}} = \sqrt{\frac{\times \quad + \quad \times}{+14}} =$$



because a higher variability is usually expected when the mean increases, and the CV is a measure that accounts for this variability. The coefficient of variation is also useful for comparing the reproducibility of different variables. CV is a relative measure free from unit of measurement. CV remains the same regardless of what units are used, because if the units are changed by a factor C, both the mean and SD change by the factor C; the CV, which is the ratio between them, remains uncharged.

Example: Compute the CV for the birth weight data when they are expressed in either grams or ounces.

Solution: in grams $\overline{X}$ = 3166.9 g, S = 445.3 g,

$$CV=100\%\times\frac{S}{\overline{X}}=100\%\times\frac{445.3}{3166.9}=14.1\%$$

If the data were expressed in ounces, $\overline{X}$ =111.71 oz, S=15.7 oz, then

$$CV = 100\% \times\frac{S}{\overline{X}}=100\%\times\frac{15.7}{111.71}=14.1\%$$

TIME (HOURS)	NO. OF STUDENTS (1)	CUM. FREQ.	MID-POINT (2)	(1)×(2)	(1)×(2)^2	d	f×d	f×d^2
10–14	8	8	12	96	1,152	-2	-16	32
15–19	28	36	17	476	8,092	-1	-28	28
20–24	27	63	22	594	13,068	0	0	0
25–29	12	75	27	324	8,748	1	12	12
30–34	4	79	32	128	4,096	2	8	16
35–39	1	80	37	37	1,369	3	3	9
Total	80			1,655	36,525		-21	97

a) Computing the arithmetic mean

i) **By Direct Method**: This method is applicable where the entire range of available values or scores of the variable has been divided into equal or unequal class intervals and the observations have been grouped into a frequency distribution on that basis. The value or score (x) of each observation is assumed to be identical with the mid-point (X_c) of the class interval to which it belongs. In such cases the mean of

For the time data the mean time spent by students for leisure activities was:

$$\overline{X} = \frac{\sum fX_c}{\sum f} = \frac{1{,}655}{80} = 20.7 hours$$

ii) **Indirect or Code Method:** This is applicable only for grouped frequency distribution with equal class interval length.

Steps:

1. Assume an arbitrary mid-point (A) as the mean of the distribution and give a code or coded value of 0.
2. Code numbers (d),..., -2, -1 and 1, 2,... are then assigned in descending and ascending order to the mid-points of the intervals running downwards from and rising progressively higher than A, respectively. The code numbers (d) of the mid-point (Xc) of a class interval may also be obtained by the following way. d = (Xc-A)/w.
3. The code numbers (d) of each class interval is multiplied by the frequency (f) of that interval and the sum of these products (Σfd) is divided by the total number of observations (n) of the sample to get the mean of the coded values;

$$\bar{X} = A + \frac{\Sigma fd}{n} \times w = 22 + \frac{-21}{80} \times 5 = 22 + 1.3125 = 20.7 hours$$

b) Computation of Median from a Grouped Frequency Distribution

The class whose cumulative frequency at least 40 is the 3rd class, i.e. 20 - 24, for this class then:

LCB = 19.5, frequency of the median class = 27, cumulative frequency next below the median class =36

$$\tilde{X} = 19.5 + \frac{40-36}{27} \times 5 = 19.5 + 0.7 = 20.2 \text{ hours}$$

In the calculation of the median from a grouped frequency table, the basic assumption is that within each class of the frequency distribution, observations are uniformly or evenly distributed over the class interval.

c) Computation of the standard deviation from a Grouped Frequency Distribution

In the calculation of the median from a frequency table, the basic assumption is that within each class of the frequency distribution, observations are uniformly or evenly distributed over the class interval. The frequencies are arranged in a cumulative frequency distribution to facilitate computations.

In a grouped frequency distribution, the SD is computed as

$$S == \sqrt{\qquad\qquad\qquad\qquad}$$

80 1)

Using the code method, the SD for equal class interval grouped frequency distribution can be calculated as

$$S = W \times \sqrt{\frac{\Sigma f(d-\bar{d})^2}{n-1}} = W \times \sqrt{\frac{n\Sigma fd^2 - (\Sigma fd)^2}{n(n-1)}}$$

Remember characteristics of standard deviation that SD of a constant is 0.

For the above data, $\Sigma fd = -21$, $\Sigma fd^2 = 97$, $W = 5$

$$S = 5 \times \sqrt{\frac{80(97) - (-21)^2}{80(80-1)}} = 5 \times 1.076 = 5.38 \text{ hours}$$

$$\sqrt{\frac{\Sigma f_i (X_{ci} - \bar{X})^2}{\Sigma f_i - 1}} = \sqrt{\frac{\Sigma f_i (X_{ci} - \bar{X})^2}{n-1}} = \sqrt{\frac{n\Sigma f_i X_{ci}^2 - (\Sigma f_i X_{ci})^2}{n(n-1)}}$$

Where Xci is the mid-point of the i^{th} class.

Example: Consider the previous data on time spend by college students for leisure activities

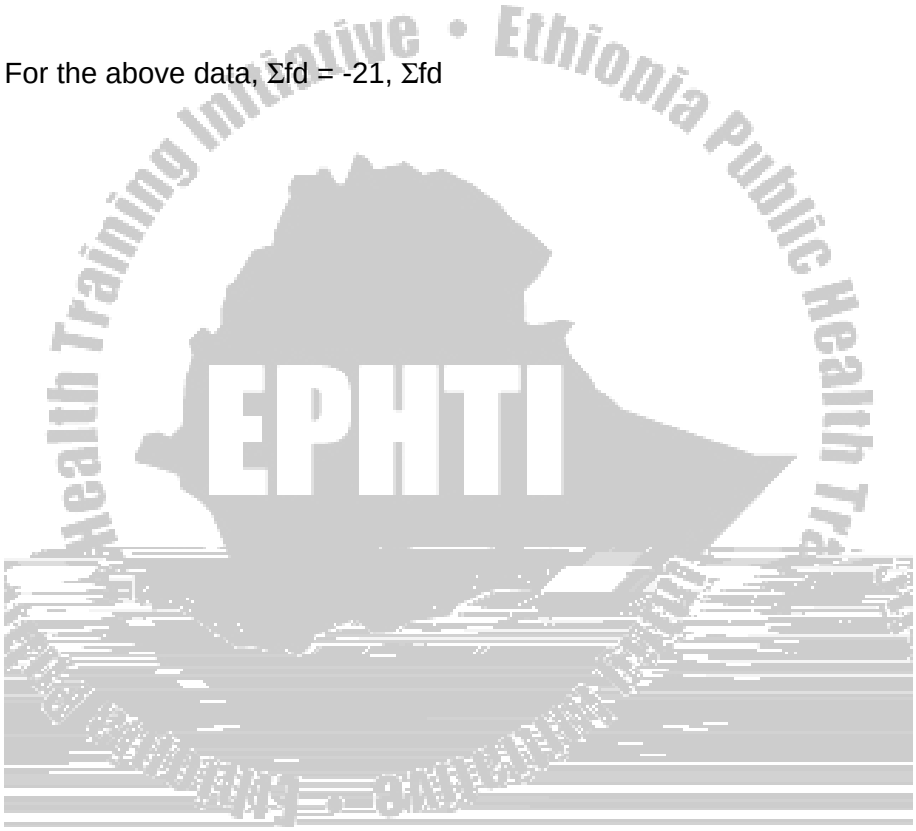
$$S = \sqrt{\frac{80(36{,}525) - (1{,}655)^2}{80(80-1)}} = 5.38 \text{ hours}$$

Using the code method, the SD for equal class interval grouped frequency distribution can be calculated as

$$S = W \times \sqrt{\frac{\Sigma f(d-\bar{d})^2}{n-1}} = W \times \sqrt{\frac{n\Sigma fd^2 - (\Sigma fd)^2}{n(n-1)}}$$

Remember characteristics of standard deviation that SD of a constant is 0.

For the above data, Σfd = -21, Σfd



2. Blood pressure levels of 60 first-year male medical students (in mm Hg)

Class limit	Frequency
90-99	2
100-109	6
110-119	17
120-129	16
130-139	12
140-149	6
150-159	1

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CHAPTER FOUR
DEMOGRAPHY AND HEALTH SERVICES STATISTICS

4.1. LEARNING OBJECTIVES

At the end of this chapter, the students will be able to:

1. Define and understand the concepts of demographic statistics
2. Identify the different methods of data collection for demographic studies
3. Understand different ratios used to describe demographic data
4. Understand fertility and mortality measures
5. Understand methods of population projection and computation of doubling time
6. Understand and compute the different indices relating to hospitals

4.2. INTRODUCTION

Definition: Demography is a science that studies human population

variation and the effect of all these on health, social, ethical, and economic conditions.

Size: is the number of persons in the population at a given time.
Example: The size of Ethiopian population in 2002 is about 65 million.

Distribution: is the arrangement of the population in the territory of the nation in geographical, residential area, climatic zone, etc.
Example: Distribution of Ethiopian population by Zone
Composition (Structure): is the distribution of a population into its various groupings mainly by age and sex.
Example: The age and sex distribution of the Ethiopian population refers to the number (%) of the population falling in each age group (at each age) by sex.

Change: refers to the increase or decline of the total population or its components. The components of change are **birth, death, and migration**.

Therefore, demographic statistics is the application of statistics to the study of human population in relation to the essential demographic variables and the source of variations of the population with respect to these variables, such as fertility, mortality and migration. It is indispensable to study these variables through other socio-economic

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De jure:- the enumeration (or count) is done according to the usual or legal place of residence

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Census Operation

The entire census operation has 3 parts (stages)

1) pre-enumeration → planning and preparatory work
2) enumeration → field work (collection of the data)
3) post-enumeration → editing, coding, compilation, tabulation, analysis, and publication of the results

Uses of a census

1) gives complete and valid picture of the population composition and characteristics
2) serves as a sampling frame
3) provides with vital statistics of the population in terms of fertility and mortality.
4) Census data are utilized in a number of ways for planning the welfare of the people

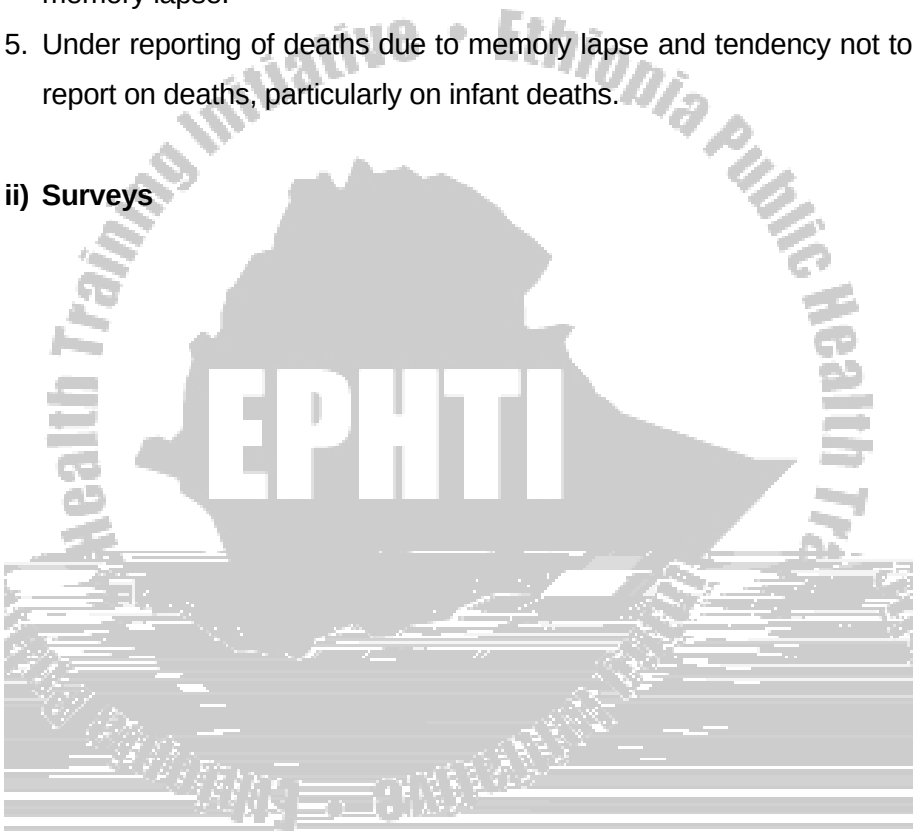
Eg. To ascertain food requirements, to plan social welfare schemes like schools, hospitals, houses, orphanages, pensions, etc.

Common errors in census data

1. Omission and over enumeration.

2. Misreporting of age due to memory lapse, preference of terminal digits, over/under estimation.
3. Overstating of the status within the occupation.
4. Under reporting of births due to problem of reference period and memory lapse.
5. Under reporting of deaths due to memory lapse and tendency not to report on deaths, particularly on infant deaths.

ii) Surveys



registered, reported to a control body and compiled centrally. A certificate is issued for every death and birth. The four main characteristics of vital registration are comprehensiveness, compulsory by law, compilation done centrally and the registration is an ongoing (continuous) process.

This system is not well developed in Ethiopia and hence cannot serve as a reliable means of getting information.

4.4 Stages in Demographic Transition

Demographic transition is a term used to describe the major demographic trends of the past two centuries. The change in population basically consists of a shift from an equilibrium condition of high birth and death rates, characteristics of agrarian societies, to a newer equilibrium in which both birth and death rates are at much lower levels. The end stage (period 3) of this demographic transition is a situation in which birth and death rates are again essentially in balance but a lower level.

The theory of demographic transition attempts to explain the changes in mortality and fertility in three different stages.

1) pre-transitional :- characterized by high an( )]TJ0 TD-0.5eTj/TT2 1 Tf0 -1.4rr0135 o((inTe -1.

persons have survived in to the older age group. This type of population is found in primitive societies and is sometimes known as expansive (type I).

2) Transitional:- characterized by high birth rate and reduced death rate, with high (rapid) growth rate ("young population"). The drop in the death rate is usually brought about by improved medical caeyvwitn-5.2(vgth)-5s]TJ0 -1.7268 TD-0

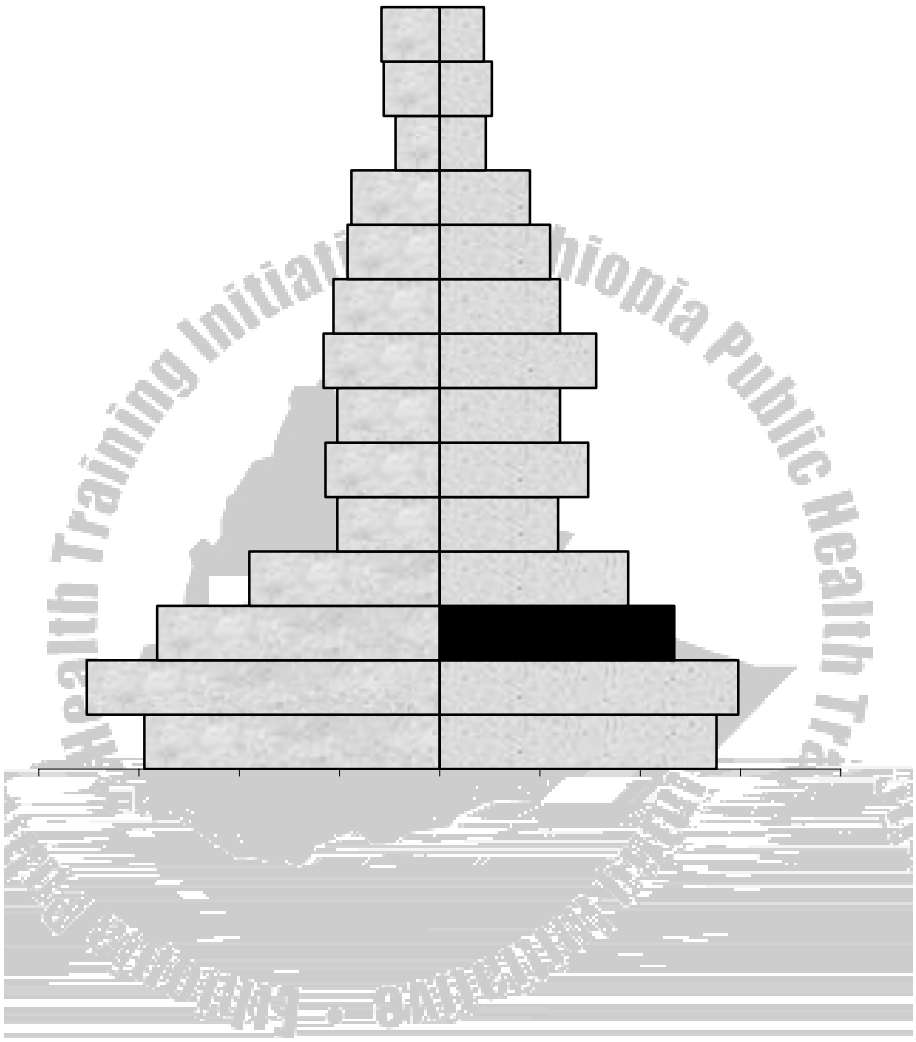
convention, males are shown on the left of the pyramid, females on the right, young persons at the bottom, and the elderly at the top.

The pyramid consists of a series of bars, each drawn proportionately to represent the percentage contribution of each age-sex group (often in five-year groupings) to the total population; that is, the total area of the bars represents 100% of the population.

A pyramid conveys at a glance the entire shape of the age structure. It shows any gross irregularities due to special past events (such as a war, epidemic or age-selective migration), fluctuations of fertility, etc.. We refer to a population as “old” or “young”, according to the relative weight of old and young age groups in the total.

Population pyramid can be constructed either using absolute number or percentages. When constructing percent population pyramids, take the percentage from the group total.

The following pyramid is drawn for data obtained from census conducted in Cheha District, Gurage Zone, July 2000.



4.5 RATIO, PROPORTION AND RATE

4. Ratio: A ratio quantifies the magnitude of one occurrence or condition in relation to another.

1. **Sex Ratio** (SR): sex ratio is defined as the total number of male population per 100 female population,

SR = $\frac{M}{F}$ x 100 where M and F are total number of male and female populations, respectively.

Sex ratios are used for purposes of comparison.

a) The balance between the two sexes
b) The variation in the sex balance at different ages
c) It is also used for detecting errors in demographic data

2. Child-Woman-Ratio (CWR): It is defined as the ratio of the number of children under 5 years of age to the number of women in the childbearing age group (usually 15-49).

CWR = P_{0-4} / P^{f}_{15-49} x 1000 = Number of children under 5 years of age per 1000 women in the child bearing age.

The child woman ratio is also known as measure of effective fertility because we are considering survivals up to the age of 4 not the dead

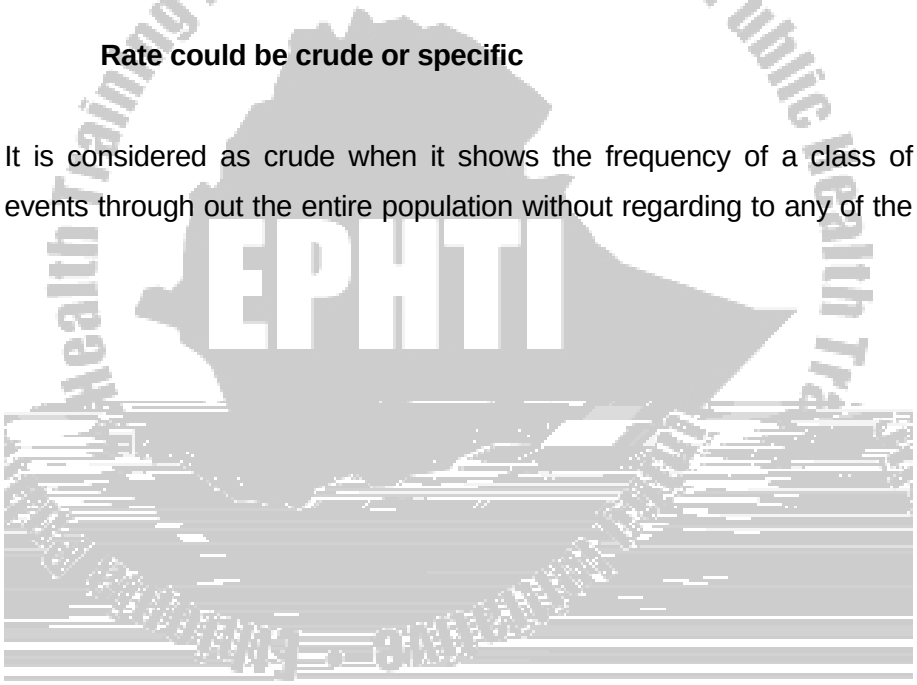
$$\text{Rate} = \frac{\text{Number of demographic events of interest}}{\text{Population at risk}} \times k$$

where K is a constant mainly a multiple of 10 (100, 1000, 10000, etc.).

Population at risk: This could be the mid-year population (population at the first of July 1), population at the beginning of the year or a more complex definition. Period for a rate is usually a year.

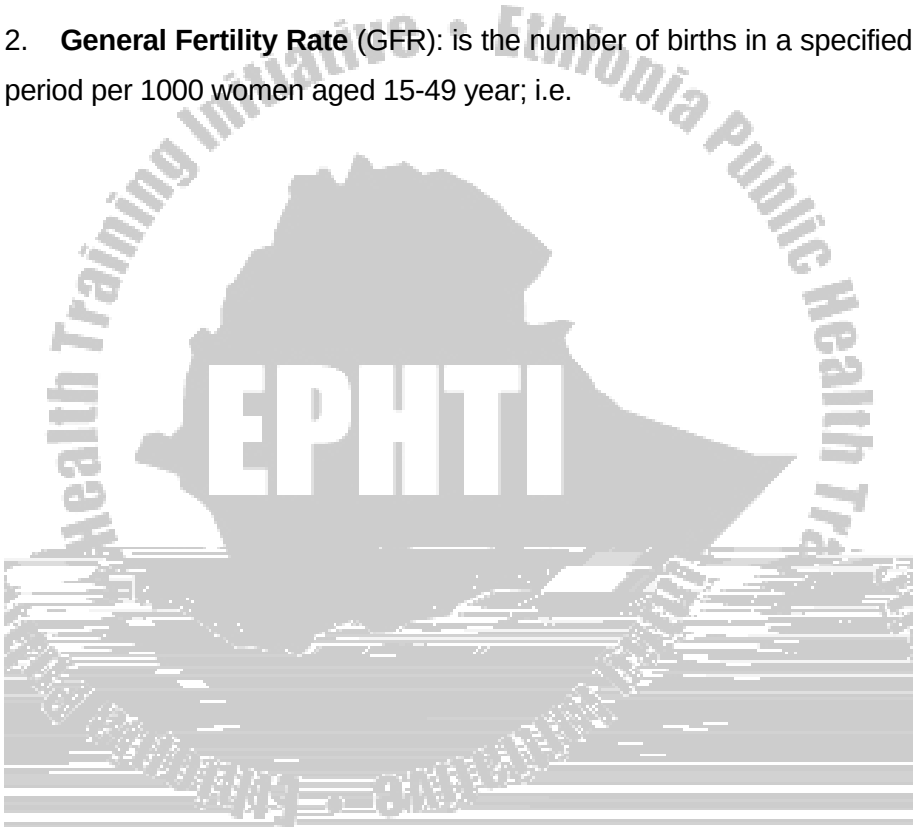
Rate could be crude or specific

It is considered as crude when it shows the frequency of a class of events through out the entire population without regarding to any of the



*(*) **Live Birth** is the complete expulsion or extraction from its mother as a product of conception irrespective of the duration of pregnancy, which after such separation show evidence of life, (like breathing, pulsation of the heart, etc.).*

2. **General Fertility Rate** (GFR): is the number of births in a specified period per 1000 women aged 15-49 year; i.e.



the end of reproduction, if there were no mortality among women of reproductive age; each woman will live up to 49 years of age, about a total of 35 years.

$$TFR = \sum_{i=15}^{49} \frac{B_i}{P_i^f} \times 1000$$ for single year classification of age

$$TFR = 5 \times \sum_{i=1}^{7} \frac{B_i}{P_i^f} \times 1000$$ for 5 year age group classification

where $B_i = B_f + B_m$ = birth of both sex at age i of mothers

P_i^f = female population at age (age interval) i.

TFR is used.1( )-5.0D a stgn216.96pulaB

The above two formulae are used if we don't have female birth and female population by age. But if we have female birth and female population by age

$$GRR = \sum_{i=15}^{49} \frac{B_i^f}{P_i^f} \times 1000 \text{ for single year age classification}$$

$$GRR = 5 \times \sum_{i=1}^{7} \frac{B_i^f}{P_i^f} \times 1000 \text{ for 5 years age grouping}$$

GRR gives the average number of daughters a synthetic cohort (group) of women would have at the end of reproduction, in the absence of mortality.

Example: If GRR = 1000 => the current generation of females of child bearing age will maintain itself on the basis of current fertility rate with out mortality.

If GRR > 1000 => no amount of reduction of deaths will enable it to escape decline sooner or later and if GRR < 1000 the reverse happens.

In the absence of birth data cross classified by age of mother at birth and sex of the new born, we can approximate GRR from TFR simply by multiplying TFR by proportion of female births on the assumption that sex ratio at birth is constant. That is, the ratio of the number of male births to the number of female births remains constant over all ages of mothers.

$$GRR=\frac{B^f}{B^t}TFR=\frac{TFR}{1+\frac{Total\,male\,births}{Total\,female\,births}}=\frac{TFR}{1+\frac{Sex\,ratio\,at\,birth}{100}}$$

Note: A rate is Birth Rate if the denominator is mid year population and it is fertility rate if the denominator is restricted to females in the child bearing age.

6. Net reproduction rate (NRR)

The main disadvantage of the gross reproduction rate is that it does not take into account the fact that not all the females will live until the end of the reproductive period. In computing the net reproduction rate, mortality of the females is taken into account. The net reproduction rate measures the extent to which the females in the childbearing age-groups are replacing themselves in the next generation. The net reproduction rate is one in a stationary population; a population which neither increases nor decreases (i.e. r = 0). In most cases, NRR is expressed per woman instead of per 1000 women.

NRR = 1 ⇒ stationary population (i.e., 1 daughter per woman)

NRR < 1⇒ declining population

NRR > 1⇒ growing population

4.7 MEASURES OF MORTALITY

1.

3. **Cause Specific Death Ratio and Rate:** A cause specific death ratio (proportionate mortality ratio) represents the percent of all deaths due to a particular cause or group of causes.

CSD ratio for cause c $= \frac{D_c}{D_t} \times 1000$, where Dc is total deaths from cause c and D_t is total deaths from all causes in a specified time period.

Cause Specific Death Rate (CSDR) is the number of deaths form cause c during a year per 1000 of the mid year population, i.e.

$$CSDR_c = \frac{\text{Total deaths from a given cause c}}{\text{Population at risk}} \times 1000$$

4. **Infant Mortality Rate** (IMR): nf.yteD(10070.2181 s)i.et Motft

$$NMR = \frac{\text{Deaths of children under 28 days of age}}{\text{Total live Births}} \times 1000$$

5. **Post - Neonatal Mortality Rate** (PNMR): Measures the risk of dying during infancy after the first 4 weeks of life, and is defined as:

$$PNMR = \frac{\text{Deaths of children aged 28 days to under one year}}{\text{Total live births}} \times 100($$

6. **Maternal Mortality Rate** (MMR): is defined as the number of deaths of mothers (Dm) due to maternal causes, i.e. complications of pregnancy, child birth, and puerperium, per 100,000 live births during a year, i.e.

$$MMR = \frac{\text{Deaths of Mothers due to maternal causes in a year}}{\text{Total live births in the same year}} \times 100{,}000$$

MMR measures the risk of dying of mothers from maternal causes.

4.8 POPULATION GROWTH AND PROJECTION

The rate of increase or decline of the size of a population by natural causes (births and deaths) can be estimated crudely by using the measures related to births and deaths in the following way:

Rate of population growth

Crude Birth Rate - Crude Death Rate = **crude rate of natural increase.** This rate is based on naturally occurring events – births and deaths. When the net effect of migration is added to the natural increase it gives what is known as total increase.

Based on the total rate of increase (r), the population (P_t) of an area with current population size of (P_o) can be projected at some time t in the short time interval (mostly not more than 5 years) using the following formula.

$P_t = P_o(1+r)^t$ **OR** $P_t = P_o \times Exp(r \times t)$ - the exponential projection formula

For example if the CBR=46, CDR=18 per 1000 population and population size of 25,460 in 1998, then

Crude rate of natural increase = 46 - 18 = 28 per 1000 = 2.8 percent per year. The net effect of migration is assumed to be zero.

The estimated population in 2003, after 5 years, using the first formula will be

$P_{2003} = P_{1998} (1 + 0.028)^t = 25{,}460(1 + 0.028)^5 = 25{,}460(1.028)^5$

$= 25{,}460(1.148) = 29{,}230$

The population of the area in 2003 will be about 29,230.

Population doubling time

The doubling time of the size of a population can be estimated based on the formula for projecting the population.

$$P_t = P_o(1+r)^t$$

From the above formula, the time at which the current population P_o will be $2 \times P_o$ can be found by:

$$2 \times P_O = P_O(1+r)^t \Rightarrow 2 = (1+r)^t \Rightarrow \log 2 =$$

$$t \times \log(1+r) \Rightarrow t = \frac{\log 2}{\log(1+r)}$$

For example, for the above community, r=0.028, then the doubling time for this population will be:

$$t = \frac{\log(2)}{\log(1+0.028)} = \frac{0.30103}{0.01199} = 25.1 \text{ years}$$

Therefore, it will take 25.1 years for the size of this population to be doubled.

A more practical approach to calculate the population doubling time is:

$2\times$ Po = Po $(1+r)^t \Rightarrow$ 2 = $(1+r)^t$ = $(e^r)^t$ = $(e)^{rt}$ (provided r is very small compared to 1)

$\Rightarrow \ln 2 = rt$

$\Rightarrow \ln 2 = rt$

$\Rightarrow 0.693 \approx 0.7 = rt$

Hence, $t = \frac{0.7}{r}$

For the above example, the doubling time(t) would be (0.7 / 0.028) = 25 years.

4.9 HEALTH SERVICES STATISTICS

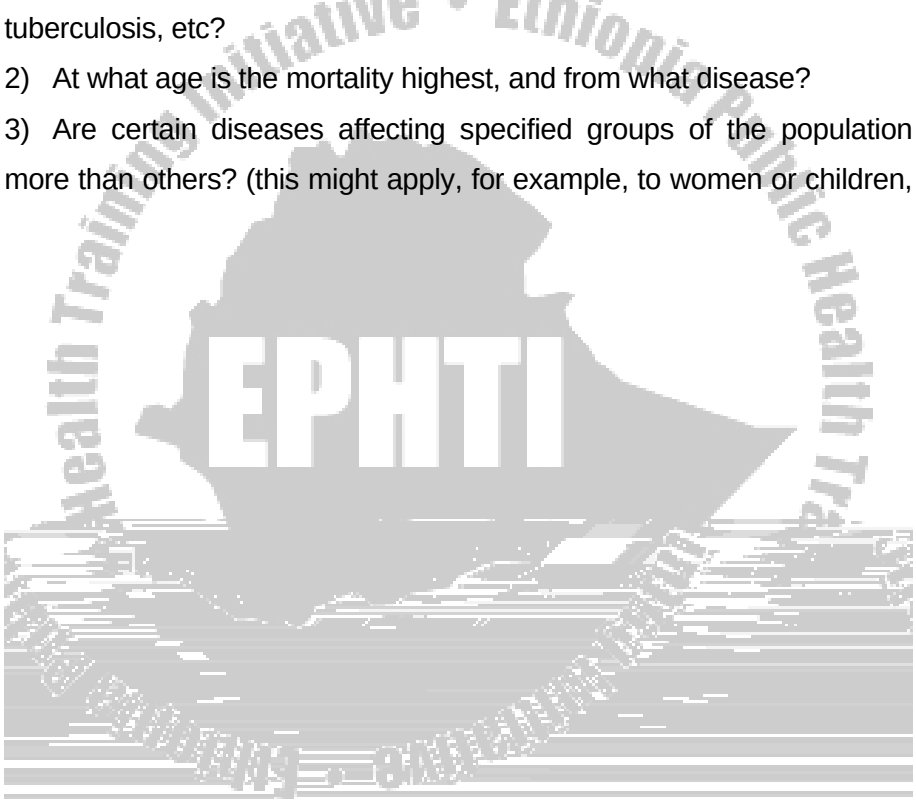
Health Service Statistics are very useful to improve the health situation

of the population of a given country. For example, the following questions could not be answered correctly unless the health statistics of a given area is consolidated and given due emphasis.

1) What is the leading cause of death in the area? Is it malaria, tuberculosis, etc?

2) At what age is the mortality highest, and from what disease?

3) Are certain diseases affecting specified groups of the population more than others? (this might apply, for example, to women or children,



administrative action

4) Determine priorities for health programmes
5) Develop procedures, definitions, techniques such as recording systems, sampling schemes, etc.
6) Promote health legislation
7) Create administrative standards of health activities
8) Determine the met and unmet health needs
9) Disseminate information on the health situation and health programmes
10) Determine success or failure of specific health programmes or undertake overall evaluation of public health work
11) Demand public support for health work

Major limitations of morbidity and mortality data from health institutions in Ethiopia include the following:

1) **Lack of completeness**: Health services at present (in 2000) cover only 47% of the population
2) **Lack of repr1.8(n)4.8(ou/TT.4(c)t(i)rv5.3(f)1.4)4. ren**

stations. Such facilities are available in hospitals.

5) **Lack of compliance with reporting:** Reports may be incomplete, not sent on time or not sent at all.

Health service utilization rates (Hospital statistics) - Indices relating to the hospital

1) **Admission rate (AR**): The number of (hospital) admissions per 1000 of the population per year

$$AR = \frac{\text{Number of Admissions in the year}}{\text{Total Population of the Catchment area}} \times 1000$$

"Admission" is the acceptance of an in-patient by a hospital.

Discharges and deaths: The annual number of discharges includes the number of patients who have left the hospital (cured, improved, etc.), the number who have transferred to another health institution, and the

$$\text{ALS} = \frac{\text{The Annual Number of Hospitalized Patient Days}}{\text{Number of Discharges and Deaths}}$$

The turnover interval is zero when the bed-occupancy rate is 100%.

A very short or negative turnover interval points to a shortage of beds, whereas a long interval may indicate an excess of beds or a defective admission mechanism.

Year	Total population of the district	No of health institutions in the district			Total number of hospital beds
		Health Station	Health Center	Hospital	
1988	400,000	14	2	1	80

During the same year, there were 14,308 discharges and deaths. The annual number of hospitalized patient days was also recorded as 28,616.

i) Calculate:

1. the health service coverage of the district
2. .the average length of stay
3. the bed occupancy rate
4. the turnover interval

ii) What do you understand from your answers in parts 1 and 4?

iii) Show that the average time that elapsed between the discharge of one patient and the admission of the next was about **one** hour.

CHAPTER FIVE
ELEMENTARY PROBABILITY AND PROBABILITY DISTRIBUTIONS

5.1 Learning Objectives

At the end of this chapter, the student will be able to:

1. Understand the concepts and characteristics of probabilities and probability distributions
2. Compute probabilities of events and conditional probabilities
3. Differentiate between the binomial and normal distributions
4. Understand the concepts and uses of the standard normal distribution

5.2 INTRODUCTION

In general, there is no completely satisfactory definition of probability. Probability is one of those elusive concepts that virtually everyone knows but which is nearly impossible to define entirely adequately.

A fair coin has been tossed 800 times. Here is a record of the number of times it came up head, and the proportion of heads in the throws already made:

The Classical Probability Concept

If there are n ***equally likely possibilities***, of which one must occur and m are regarded as favourable, or as a "success," then the probability of a "success" is m/n.

Example: What is the probability of rolling a 6 with a *well-balanced die*?

In this case, m=1 and n=6, so that the probability is 1/6 = 0.167

Definitions of some terms commonly encountered in probability

Experiment

Let A = the event an odd number turns up, A = (1,3,5)

Let B = the event a 1,2 or 3 turns up; B = (1,2,3)

Let C = the event a 2 turns up, C= (2)

i) Find Pr (A); Pr (B) and Pr (C)

Pr(A) = Pr(1) + Pr(3) + Pr(5) = 1/6+1/6+ 1/6 = 3/6 = 1/2

Pr(B) = Pr(1) + pr(2) + Pr(3) = 1/6+1/6+1/6 = 3/6 = ½

Pr (C) = Pr(2) = 1/6

ii) Are A and B; A and C; B and C mutually exclusive?

- A and B are not mutually exclusive. Because they have the elements 1 and 3 in common

- similarly, B and C are not mutually exclusive. They have the element 2 in common.

- A and C are mutually exclusive. They don't have any element in common

When A and B are not mutually exclusive pr(A or B) = Pr(A) + Pr(B) cannot be used. The reason is that in vc-5.7(a)]TJ0 -1.7p

+ Pr(B) – Pr(A and B). The formula considered earlier for mutually exclusive events is a special case of this, since pr(A and B) = 0.

Eg. Of 200 seniors at a certain college, 98 are women, 34 are majoring in Biology, and 20 Biology majors are women. If one student is chosen at random from the senior class, what is the probability that the choice will be either a Biology major or a women).

Pr (Biology major or woman) = Pr (Biology major) + Pr(woman) - Pr (Biology major and woman) =34/200 + 98/200 - 20/200 = 112/200 = .56

5.4 Conditional probabilities and the multiplicative law

Sometimes the chance a particular event happens depends on the outcome of some other event. This applies obviously with many events that are spread out in time.

Eg. The chance a patient with some disease survives the next year depends on his having survived to the present time. Such probabilities are called conditional.

The notation is Pr(B/A), which is read as “the probability event B occurs given that event A has already occurred .”

Let A and B be two events of a sample space S. The conditional probability of an event A, given B, denoted by Pr (A/B)= P(A n B) / P(B) , P(B) ≠ 0.

Similarly, P(B/A) = P(A n B) / P(A) , P(A) ≠ 0. This can be taken as an alternative form of the multiplicative law.

Eg. Suppose in country X the chance that an infant lives to age 25 is .95, whereas the chance that he lives to age 65 is .65. For the latter, it is understood that to survive to age 65 means to survive both from birth to age 25 and from age 25 to 65. What is the chance that a person 25 years of age survives to age 65?

a

Notation	Event	Probability
A	Survive birth to age 25	.95
A and B	Survive both birth to age 25 and age 25 to 65	.65
B/A	Survive age 25 to 65 given survival to age 25	?

Then, Pr(B/A) = Pr(A n B) / Pr(A) = .65/.95 = .684 . That is, a person aged 25 has a 68.4 percent chance of living to age 65.

Independent Events

Often there are two events such that the occurrence or nonoccurrence of one does not in any way affect the occurrence or

nonoccurrence of the other. This defines independent events. Thus, if events A and B are independent, Pr(B/A) = P(B); Pr(A/B) = P(A).

Eg. 1) A classic example is n tosses of a coin and the chances that on each toss it lands heads. These are independent events. The chance of heads on any one toss is independent of the number of previous heads. No matter how many heads have already been observed, the chance of heads on the next toss is ½.

Eg 2) A similar situation prevails with the sex of offspring. The chance of a male is approximately ½. Regardless of the sexes of previous offspring, the chance the next child is a male is still ½.

with independent events, the multiplicative law becomes :

$$Pr(A \text{ and } B) = Pr(A)\, Pr(B)$$

Hence, $Pr(A) = Pr(A \text{ and } B) / Pr(B)$, where $Pr(B) \neq 0$

$Pr(B) = Pr(A \text{ and } B) / Pr(A)$, where $Pr(A) \neq 0$

EXERCISE

Consider the drawing of two cards one after the other from a deck of 52 cards. What is the probability that both cards will be spades?

a) with replacement

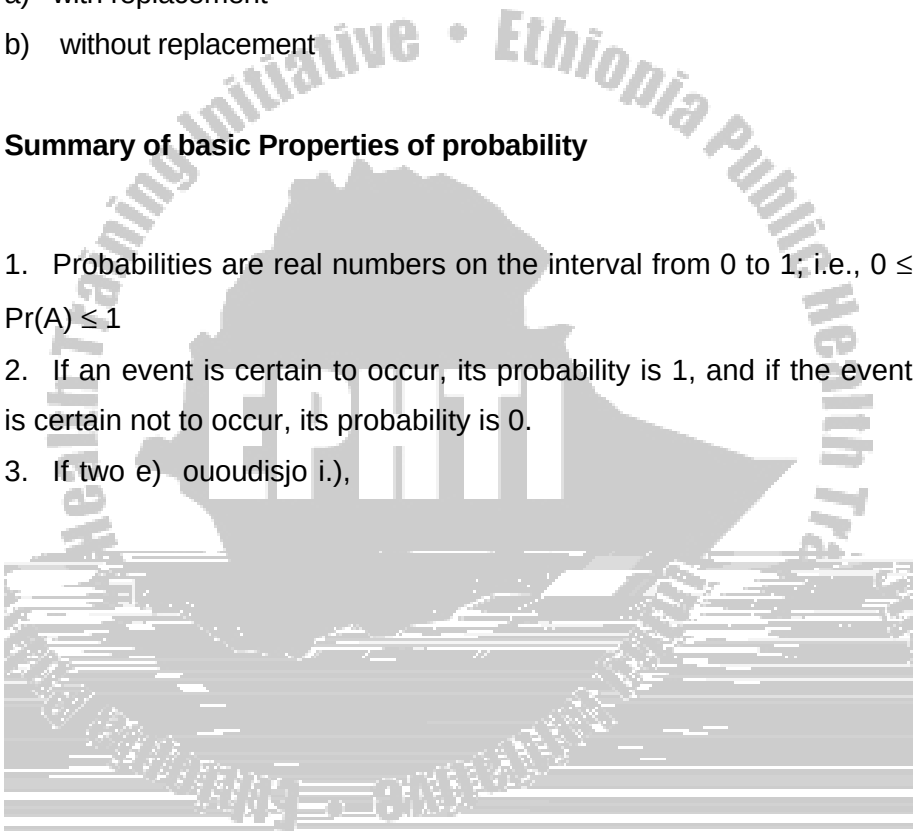
b) without replacement

Summary of basic Properties of probability

1. Probabilities are real numbers on the interval from 0 to 1; i.e., $0 \leq Pr(A) \leq 1$

2. If an event is certain to occur, its probability is 1, and if the event is certain not to occur, its probability is 0.

3. If two e) ououdisjo i.),



5.5 Random variables and probability distributions

Usually numbers can be associated with the outcomes of an experiment. For example, the number of heads that come up when a coin is tossed four times is 0, 1,2,3 or 4. Sometimes, we may find a situation where the elements of a sample space are categories. In such cases, we can assign numbers to the categories .

Eg. There are 2,500 men and 2000 women in a senior class. Assume a person is randomly selected .

Sample space	Number assigned
Man	1
Woman	2

5.5

Definition: A random variable whose values form a continuum (i.e., have no gaps) such that ranges of values occur with specified probabilities is a continuous random variable.

The values taken by a discrete random variable and its associated probabilities can be expressed by a rule, or relationship that is called a *probability mass (density) function.*

Definition: A *probability distribution (mass function*)is a mathematical relationship, or rule, that assigns to any possible value of a discrete random variable X the probability P(X = xi). This assignment is made for all values xi that have positive probability. The probability distribution can be displayed in the form of a table giving the values and their associated probabilities and/or it can be expressed as a mathematical formula giving the probability of all possible values.

General rules which apply to any probability distribution:

1. Since the values of a probability distribution are probabilities, they must be numbers in the interval from 0 to 1.
2. Since a random variable has to take on one of its values, the sum of all the values of a probability distribution must be equal to 1.

Eg. Toss a coin 3 times. Let x be the number of heads obtained. Find the probability distribution of x .

f (x) = Pr (X = xi) , i = 0, 1, 2, 3.

Pr (x = 0) = 1/8 …….. TTT

Pr (x = 1) = 3/8 …….. HTT THT TTH

Pr (x = 2) = 3/8 ……..HHT THH HTH

Pr (x = 3) = 1/8 …… HHH

Probability distribution of X.

X = xi	0	1	2	3
Pr(X=xi)	1/8	3/8	3/8	1/8

The required conditions are also satisfied. i) f(x) ≥

$$E(X) = \mu = \sum_{i=1}^{n} x_i P(X = x_i)$$

Where the x_i's are the values the random variable assumes with positive probability

Example: Consider the random variable representing the number of episodes of diarrhoea in the first 2 years of life. Suppose this random variable has a probability mass function as below

R	0	1	2	3	4	5	6
P(X = r)	.129	.264	.271	.185	.095	.039	.017

What is the expected number of episodes of diarrhoea in the first 2 years of life?

E(X) = 0(.129)+1(.264)+2(.271)+3(.185)+4(.095)+5(.039)+6(.017)= 2.038

Thus, on the average a child would be expected to ha.3(1to ha.3(1to ha.3(1to ha.3(1to ha.3to h

from the expected value by its respective probability and summing overall the values that have positive probability.

Definition: The variance of a discrete random variable denoted by X is defined by

$$V(X) = \sigma^2 = \sum_{i=1}^{k}(x_i - \mu)^2 P(X = x_i) = \sum_{i=1}^{k} x_i^2 P(X = x_i) - \mu^2$$

Where the X_i's are the values for which the random variable takes on positive probability. The SD of a random variable X, denoted by SD(X) or σ is defined by square root of its variance.

Example: Compute the variance and SD for the random variable representing number of episodes of diarrhoea in the first 2 years of life.

$E(X) = \mu = 2.04$

$\sum_{i=1}^{n} x_i P(X = x_i)$ = $0^2(.129) + 1^2(.264) + 2^2(.271) + 3^2(.185) + 4^2(.095) + 5^2(.039) + 6^2(0.017) = 6.12$

Thus, $V(X) = 6.12 - (2.04)^2 = 1.967$ and the SD of X is

$$\sigma = \sqrt{1.967} = 1.402$$

ii. THE BINOMIAL DISTRIBUTION

Binomial assumptions:

1) The same experiment is carried out n times (n trials are made).

2) Each trial has two possible outcomes (usually these outcomes are called " success" and " failure". Note that a successful outcome does not imply a good one, nor failure a bad outcome. If P is the probability of success in one trial, then , 1-p is the probability of failure.

3) The result of each trial is independent of the result of any other trial.

Definition: If the binomial assumptions are satisfied, the probability of r successes in n trials is:

$$r)r \quad \binom{n}{r} P^r (1 \quad P)^{n \ r}$$

If the true proportion of events of interest is P, then in a sample of size n the mean of the binomial distribution is **n×p** and the standard deviation is $\sqrt{np(1-p)}$

Example: Assume that, when a child is born, the probability it is a girl is ½ and that the sex of the child does not depend on the sex of an older sibling.

A) Find the probability distribution for the number of girls in a family with 4 children.

B) Find the mean and the standard deviation of this distribution.

$f(x) = p(X=r) = {}_4C_r\ (1/2)^r\ (1/2)^{4-r}$; X= 0 ,1, 2, 3, 4.

A) Probability distribution

X	0	1	2	3	4
P(X=r)	1/16	4/16	6/16	4/16	1/16

B) Mean = nP = 4 x 1/2 = 2

Standard deviation = $\sqrt{nP(1-P)}$ = $\sqrt{4\times 1/2\times 1/2}$ = $\sqrt{1}$ = 1

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weight). The normal distribution is a theoretical, continuous probability distribution whose equation is:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad \text{for } -\infty < x < +\infty$$

The area that represents the probability between two points c and d on abscissa is defined by:

$$P(c < X < d) = \int_c^d \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

The important characteristics of the Normal Distribution are:

1) It is a probability distribution of a continuous variable. It extends from minus infinity(-∞) to plus infinity (+∞).
2) It is unimodal, bell-shaped and symmetrical about x = u.
3) It is determined by two quantities: its mean (**μ**) and SD (**σ**).

possible values so that the probability of any specific value is zero.

5. An observation from a normal distribution can be related to a standard normal distribution (**SND**) which has a published table. Since the values of μ and σ will depend on the particular problem in hand and tables of the normal distribution cannot be published for all values of μ and σ, calculations are made by referring to the standard normal distribution which has $\mu = 0$ and $\sigma = 1$. Thus an observation x from a normal distribution with mean μ and standard deviation σ can be related to a Standard normal distribution by calculating :

   SND $= Z = (x - \mu) / \sigma$

Area under any Normal curve

To find the area under a normal curve (with mean μ and standard deviation σ) between x=a and x=b, find the Z scores corresponding to a and b (call them Z_1 and Z_2) and then find the area under the standard normal curve between Z_1 and Z_2 from the published table.

Z- Scores

Assume a distribution has a mean of 70 and a standard deviation of 10.

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From the symmetry properties of the stated normal distribution,

$P(Z \leq -x) = P(Z \geq x) = 1 - P(z \leq x)$

Example1: Suppose a borderline hypertensive is defined as a person whose DBP is between 90 and 95 mm Hg inclusive, and the subjects are 35-44-year-old males whose BP is normally distributed with mean 80 and variance 144. What is the probability that a randomly selected person from this population will be a borderline hypertensive?

Solution: Let X be DBP, $X \sim N(80, 144)$

$$P(90 < X < 95) = P\left(\frac{90-80}{12} < \frac{x-\mu}{\sigma} < \frac{95-80}{12}\right) = P(0.83 < z < 1.25)$$

$$= P(Z < 1.25) - P(Z < 0.83) = 0.8944 - 0.7967 = 0.098$$

Thus, approximately 9.8% of this population will be borderline hypertensive.

Example2: Suppose that total carbohydrate intake in 12-14 year old males is normally distributed with mean 124 g/1000 cal and SD 20 g/1000 cal.

a) What percent of boys in this age range have carbohydrate intake above 140g/1000 cal?

b) What percent of boys in this age range have carbohydrate intake below 90g/1000 cal?

Solution: Let X be carbohydrate intake in 12-14-year-old males and

$X \sim N(124, 400)$

a) $P(X > 140) = P(Z > (140-124)/20) = P(Z > 0.8)$

$= 1 - P(Z < 0.8) = 1 - 0.7881 = 0.2119$

b) $P(X < 90) = P(Z < (90-124)/20) = P(Z < -1.7)$

$= P(Z > 1.7) = 1 - P(Z < 1.7) = 1 - 0.9554 = 0.0446$

b. Exercises

1. Assume that among diabetics the fasting blood level of glucose is approximately normally distribute with a mean of 105 mg per 100 ml and SD of 9 mg per 100 ml.

a) What proportions of diabetics have levels between 90 and 125 mg per 100 ml?

b) What proportions of diabetics have levels below 87.4 mg per 100 ml?

c) What level cuts of the lower 10% of diabetics?

d) What are the two levels which encompass 95% of diabetics?

Answers a) 0.9393b) 0.025 c) 93.48 mg per 100 ml

d) X_1 = 87.36 mg per 100 ml and X_2 = 122.64 mg per 100 ml

2. Among a large group of coronary patients it is found that their serum cholesterol levels approximate a normal distribution. It was found that 10% of the group had cholesterol levels below 182.3 mg per 100 ml where as 5% had values above 359.0 mg per 100 ml. What is the mean and SD of the distribution?

Answers: mean = 260 ml per 100 ml and standard deviation = 60 mg per 100 ml

3. Answer the following questions by referring to the table of the standard normal distribution.

a) If Z = 0.00, the area to the right of Z is _____.

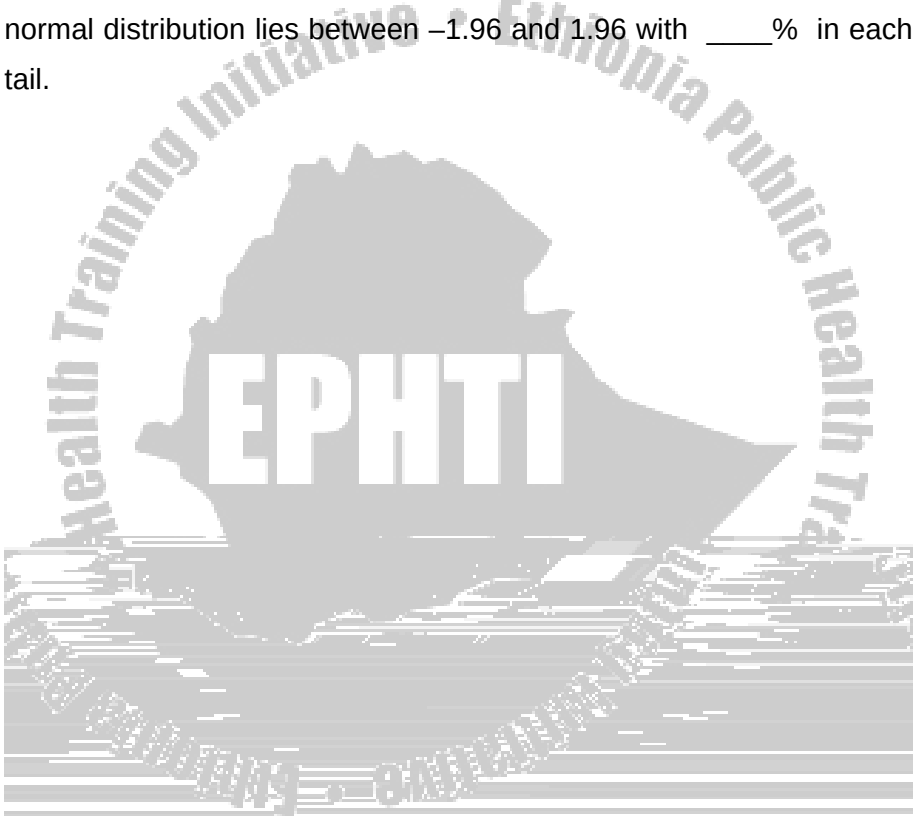
b) If Z = 0.10, the area to the right of Z is _____.

c) If Z = 0.10, the area to the left of Z is _____.

d) If Z = 1.14, the area to the right of Z is _____.

e) If Z = -1.14, the area to the left of Z is _____.

If Z = 1.96, the area to the right of Z is_____ and the area to the left of Z = - 1.96 is _____. Thus, the central 95% of the standard normal distribution lies between –1.96 and 1.96 with ____% in each tail.



CHAPTER SIX

SAMPLING METHODS

6.1 LEARNING OBJECTIVES

At the end of this chapter, the students will be able to:

1. Define population and sample and understand the different sampling terminologies
2. Differentiate between probability and Non-Probability sampling methods and apply different techniques of sampling
3. Understand the a

Advantages of samples

- cost - sampling saves time, labour and money
- quality of data - more time and effort can be spent on getting reliable data on each individual included in the sample.
  - Due to the use of better trained personnel, more careful supervision and processing a sample can actually produce precise results.

If we have to draw a sample, we will be confronted with the following questions:

a) What is the group of people (population) from which we want to draw a sample?
b) How many people do we need in our sample?
c) How will these people be selected?

Apart from persons, a population may consist of mosquitoes, villages, institutions, etc.

6.3 Common terms used in sampling

Reference population (also called source population or target population) -

would like to generalize the results of the study, and from which a representative sample is to be drawn.

Study or sample population - the population included in the sample.

Sampling unit - the unit of selection in the sampling process

Study unit - the unit on which information is collected.

- the sampling unit is not necessarily the same as the study unit.
- if the objective is to determine the availability of latrine, then the study unit would be the household; if the objective is to determine the prevalence of trachoma, then the study unit would be the individual.

Sampling frame - the list of all the units in the reference population, from which a sample is to be picked.

Sampling fraction (Sampling interval) - the ratio of the number of units in the sample to the number of units in the reference population **(n/N)**

6.4 Sampling methods (Two broad divisions)

6.4.1 Non-probability Sampling Methods

- Used when a sampling frame does not exist
- No random selection (unrepresentative of the given population)
- Inappropriate if the aim is to measure variables and generalize findings obtained from a sample to the population.

Two such non-probability sampling methods are:

A) Convenience sampling: is a method in which for convenience sake the study units that happen to be available at the time of data collection are selected.

B) Quota sampling: is a method that ensures that a certain number of sample units from different categories with specific characteristics are represented. In this method the investigator interviews as many people in each category of study unit as he can find until he has filled his quota.

Both the above methods do not claim to be representative of the entire population.

6.4.2 Probability Sampling methods

- A sampling frame exists or can be compiled.
- Involve random selection procedures. All units of the population should have an equal or at least a known chance of being included in the sample.
- Generalization is possible (from sample to population)

A) Simple random sampling (SRS)

- This is the most basic scheme of random sampling.
- Each unit in the sampling frame has an equal chance of being selected
- representativeness of the sample is ensured.

However, it is costly to conduct SRS. Moreover, minority subgroups of interest in the population my not be present in the sample in sufficient numbers for study.

To select a simple random sample you need to:

- Make a numbered list of all the units in the population from which you want to draw a sample.
  - Each unit on the list should be numbered in sequence from 1 to

- Select the required number of study units, using a “lottery” method or a table of random numbers.

"**Lottery” method**: for a small population it may be possible to use the “lottery” method: each unit in the population is represented by a slip of paper, these are put in a box and mixed, and a sample of the required size is drawn from the box.

Table of random numbers: if there are many units, however, the above technique soon becomes laborious. Selection of the units is greatly facilitated and made more accurate by using a set of random numbers in which a large number of digits is set out in random order. The property of a table of random numbers is that, whichever way it is read, vertically in columns or horizontally in rows, the order of the digits is random. Nowadays, any scientific calculator has the same facilities.

B) Systematic Sampling

Individuals are chosen at regular intervals (for example, every k^{th}) from the sampling frame. The first unit to be selected is taken at random from among the first k units. For example, a systematic sample is to be selected from 1200 students of a school. The sample size is decided to be 100. The sampling fraction is: 100 /1200 = 1/12. Hence, the sample interval is 12.

The number of the first student to be included in the sample is chosen randomly, for example by blindly picking one out of twelve pieces of paper, numbered 1 to 12. If number 6 is picked, every twelfth student will be included in the sample, starting with student number 6, until 100 students are selected. The numbers selected would be 6,18,30,42,etc.

Merits

- Systematic sampling is usually less time consuming and easier

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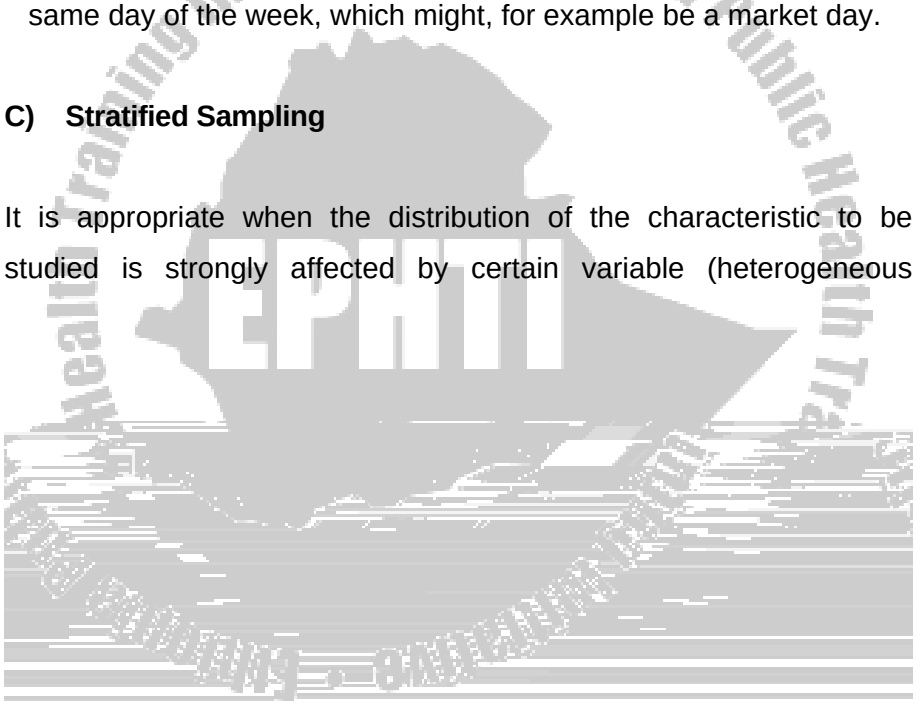
Examples

- List of married couples arranged with men's names alternatively with the women's names (every 2nd, 4th , etc.) will result in a sample of all men or women).
- If we want to select a random sample of a certain day (sampling fraction on which to count clinic attendance, this day may fall on the same day of the week, which might, for example be a market day.

C) Stratified Sampling

It is appropriate when the distribution of the characteristic to be studied is strongly affected by certain variable (heterogeneous



Merit

- The representativeness of the sample is improved. That is, adequate representation of minority subgroups of interest can be ensured by stratification and by varying the sampling fraction between strata as required.

DEMERIT

- Sampling frame for the entire population has to be prepared separately for each stratum.

D)

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It is preferable to select a large number of small clusters rather than a small number of large clusters.

Merit

A list of all the individual study units in the reference population is not required. It is sufficient to have a list of clusters.

Demerit

It is based on the assumption that the characteristic to be studied is uniformly distributed throughout the reference population, which may not always be the case. Hence, sampling error is usually higher than for a simple random sample of the same size.

E) Multi-stage sampling

This method is appropriate when the reference population is large and widely scattered . Selection is done in stages until the final sampling unit (eg., households or persons) are arrived at. The primary sampling unit (PSU) is the sampling unit (usually large size) in the first sampling stage. The secondary sampling unit (SSU) is the sampling unit in the second sampling stage, etc.

Example - The PSUs could be *kebeles* and the SSUs could be households.

Merit -

which is a form of random error. Sampling error can be minimized by increasing the size of the sample. When n = N $\Rightarrow$ sampling error = 0

6.5.2 Non-sampling error (bias)

It is a type of systematic error in the design or conduct of a sampling procedure which results in distortion of the sample, so that it is no longer representative of the reference population. We can eliminate or reduce the non-sampling error (bias) by careful design of the sampling procedure and not by increasing the sample size.

Example: If you take male students only from a student dormitory in Ethiopia in order to determine the proportion of smokers, you would result in an overestimate, since females are less likely to smoke. Increasing the number of male students would not remove the bias.

- There are several possible sources of bias in sampling (eg., accessibility bias, volunteer bias, etc.)
- The best known source of bias is non response. It is the failure to obtain information on some of the subjects included in the sample to be studied.
- Non response results in significant bias when the following two conditions are both fulfilled.

- When non-respondents constitute a significant proportion of the sample (about 15% or more)
- When non-respondents differ significantly from respondents.

- There are several ways to deal with this problem and reduce the possibility of bias:
  a) Data collection tools (questionnaire) have to be pre-tested.
  b) If non response is due to absence of the subjects, repeated attempts should be considered to contact study subjects who were absent at the time of the initial visit.
  c) To include additional people in the sample, so that non-respondents who were absent during data collection can be replaced (make sure that their absence is not related to the topic being studied).

NB: The number of non-responses should be documented according to type, so as to facilitate an assessment of the extent of bias introduced by non-response.

CHAPTER SEVEN

ESTIMATION

7.1 Learning objectives

At the end of this chapter the student will be able to:

1. Understand the concepts of sample statistics and population parameters
2. Understand the principles of sampling distributions of means and proportions and calculate their standard errors
3. Understand the principles of estimation and differentiate between point and interval estimations
4. Compute appropriate confidence intervals for population means and proportions and interpret the findings
5. Describe methods of sample size calculation for cross – sectional studies

7.2 Introduction

In this chapter the concepts of sample statistics and population parameters are described. The sample from a population is used to provide the estimates of the population parameters. The standard error, one of the most important concepts in statistical inference, is

introduced. Methods for calculating confidence intervals for population means and proportions are given. The importance of the normal distribution (Z distribution) is stressed throughout the chapter.

7.3 Point Estimation

Definition: A parameter is a numerical descriptive measure of a population (μ is an example of a parameter). A statistic is a numerical descriptive measure of a sample ($\overline{X}$ is an example of a statistic).

To each sample statistic there corresponds a population parameter. We use $\overline{X}$, S^2, S , p, etc. to estimate μ, σ^2, σ, P (or π), etc.

Sample statistic	**Corresponding population parameter**
$\overline{X}$ (sample mean)	μ (population mean)
S^2 (sample variance)	σ^2 (population variance)
S (sample Standard deviation)	σ(population standard deviation)
p (sample proportion)	P or π (Population proportion)

We have already seen that the mean $\overline{X}$ of a sample can be used to estimate μ.This does not, of course, indicate that the mean of every sample will equal the population mean.

Definition: A point estimate of some population parameter O is a single

value Ô of a sample statistic.

Eg. The mean survival time of 91 laboratory rats after removal of the thyroid gland was 82 days with a standard deviation of 10 days (assume the rats were randomly selected).

observations in the population.

4) The result is a series of means of samples of size n. If each mean in the series is now treated as an individual observation and arrayed in a frequency distribution, one determines the sampling distribution of means of samples of size n.

Because the scores ($\overline{X}$ s) in the sampling distribution of means are themselves means (of individual samples), we shall use the notation $\sigma_{\overline{X}}$ for the standard deviation of the distribution. The standard deviation of the sampling distribution of means is called the standard error of the mean.

Eg.
- Obtain repeat samples of 25 from a large population of males.
- Determine the mean serum uric acid level in each sample by replacing the 25 observations each time.
- Array the means into a distribution.
- Then you will generate the sampling distribution of mean serum uric acid levels of samples of size 25.

Properties

1. The mean of the sampling distribution of means is the same as the population mean, μ.

2. The SD of the sampling distribution of means is $\sigma / \sqrt{n}$.

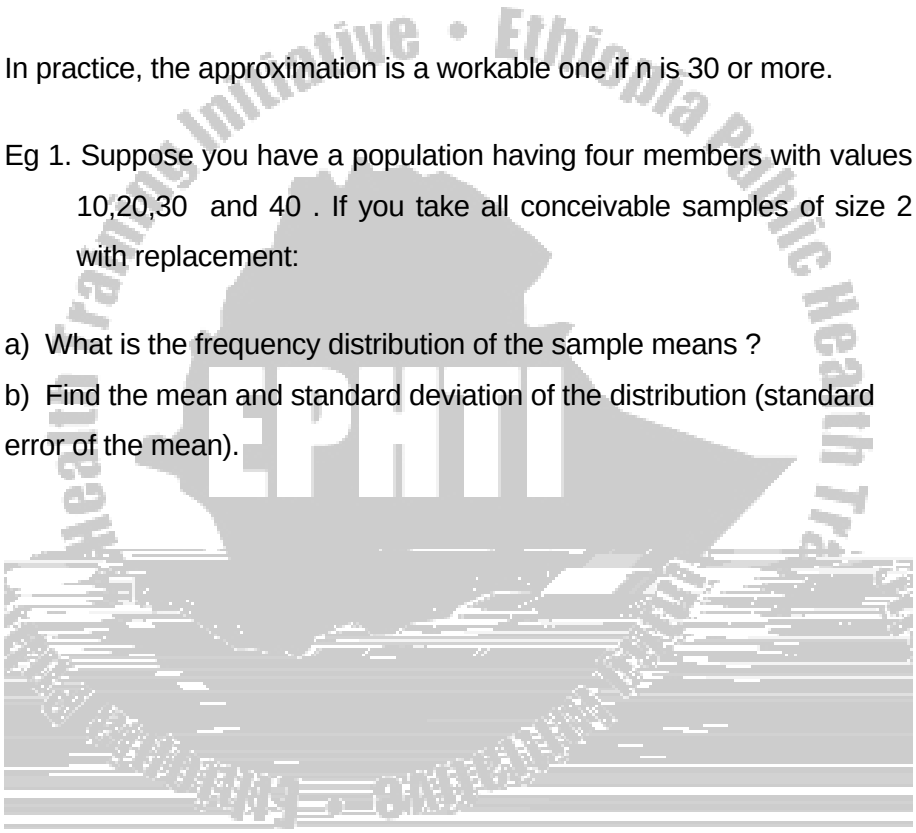
3. The shape of the sampling distribution of means is approximately a normal curve, regardless of the shape of the population distribution and provided n is large enough (Central limit theorem).

In practice, the approximation is a workable one if n is 30 or more.

Eg 1. Suppose you have a population having four members with values 10,20,30 and 40 . If you take all conceivable samples of size 2 with replacement:

a) What is the frequency distribution of the sample means ?

b) Find the mean and standard deviation of the distribution (standard error of the mean).



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(that is, the standard error of the mean is equal to the population standard deviation divided by the square root of the sample size)

Answers to example 2

a) $\mu = \sum xi / N = (10 + 20 + 30 + 40) / 4 = 25$

b) $\sigma^2 = \sum(xi - \mu)^2 / N = (225 + 25 + 25 + 225) / 4 = 125$

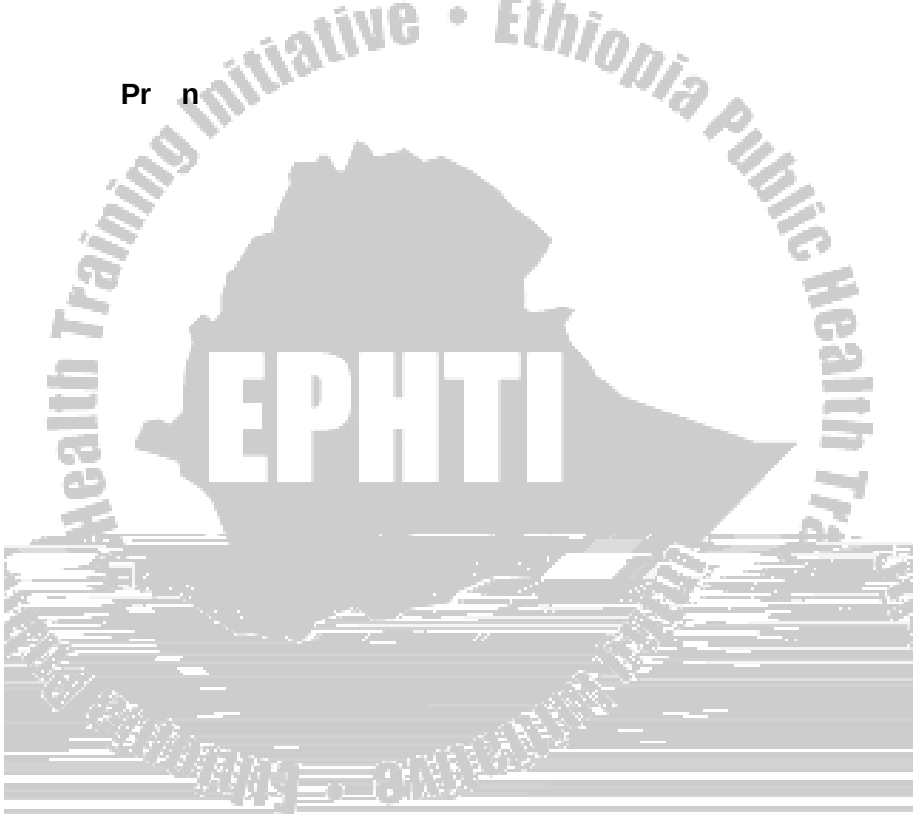
Hence, $\sigma = \sqrt{125} = 11.18$ and σ_x (standard error) $= \sigma_x / \sqrt{n} = 11.180 / 1.414 = 7.9$

7.5 Interval Estimation (large samples)

A point estimate does not give any indication on how far away the parameter lies. A more useful method of estimation is to compute an interval which has a high probability of containing the parameter.

This is merely a shorthand algebraic statement that 95% of the standard normal curve lies between + 1.96 and –1.96. If one chooses the sampling distribution of means (a normal curve with mean μ and standard deviation $\sigma / \sqrt{n}$), then ,

Pr n



for each is determined, then it is expected that 95% of these computed intervals will contain the population mean (μ) .

Clearly, there appears to be no rationale (logical basis) for taking repeated samples of size n and determine the corresponding confidence intervals. However, the knowledge of the properties of these sampling distributions of means (if one hypothetically obtained these repeated samples) permits one to draw a conclusion based upon one sample and this was shown repeatedly in the previous sections.

From the above definition of Confidence interval (C.I.), the widely used definition is derived. That is, when one claims $\overline{X}$ **± 1.96 ($\sigma/\sqrt{n}$)** as the limits on μ, there is a 95% chance that the statement is correct (that μ is contained within the interval).

If more than 95% certainty regarding the population mean - say, a 99% C.I. were desired, the only change needed is to use ±2.58 (the point enclosing 99% of the standard normal curve), which gives $\overline{X}$ **± 2.58 ($\sigma/\sqrt{n}$) .**

Eg 1. The mean reading speed of a random sample of 81 adults is 325 words per minute. Find a 90% C.I. For the mean reading speed of all adults (μ) if it is known that the standard deviation for all adults is 45 words per minute.

Given

n = 81

σ = 45

x = 325

Z = ± 1.64 (the point enclosing 90% of the standard normal curve)

A 90% C.I. for μ is $\bar{x}$ **± 1.64 (σ /√n) = 325 ± (1.64 x 5) = 325 ± 8.2**

= (316.8, 333.2)

Therefore, A 90% CI. For μ is 316.8 to 333.2words per minute.

Eg 2. A random sample of 100 drug-treated patients has a mean survival time of 46.9 months. If the SD of the population is 43.3 months, find a 95% confidence interval for the population mean.

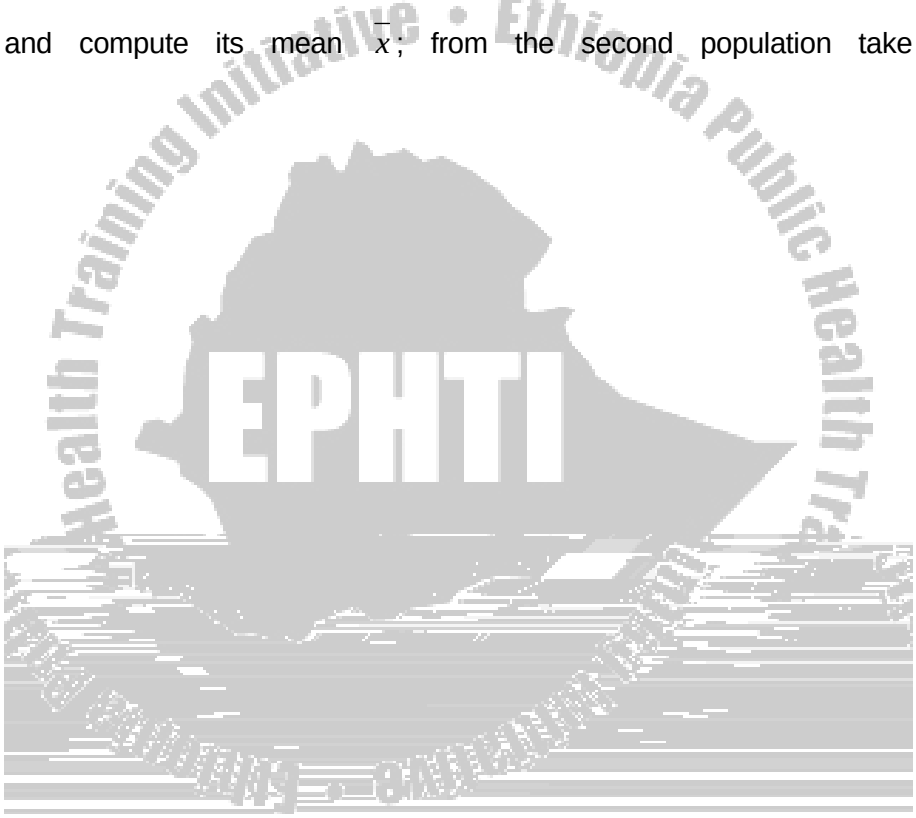
(The population consists of survival times of cancer patients who have been treated with a new drug)

46.9 ± (1.96) (43.3 /√100) = 46.9 ± 8.5 = (38.4 to 55.4 months)

Hence, there is 95% certainty that the limits (38.4 , 55.4) embrace the mean survival times in the population from which the sample arose.

7.5.2 Confidence interval for the difference of means

Consider two different populations. The first population (X) has mean μ_x and standard deviation σ_x, the second (Y) has mean μ_y and standard deviation σ_y. From the first population take a sample of size n_x and compute its mean $\bar{x}$; from the second population take



3) The sampling distribution is normal if both populations are normal, and is approximately normal if the samples are large enough (even if the populations aren't normal). In practice, it is assumed that the sampling distribution of differences of means is normal if both n_x and n_y are ≥30.

A formula for C.I is found by solving $Z = \{(\bar{x} - \bar{y}) - (\mu_x - \mu_y)\} / \sigma_{(\bar{x} - \bar{y})}$ for $\mu_x - \mu_y$; hence C.I. for the difference of means is $(\bar{x} - \bar{y}) \pm Z.\sigma_{(\bar{x} - \bar{y})}$

Eg1. If a random sample of 50 non-smokers have a mean life of 76 years with a standard deviation of 8 years, and a random sample of 65 smokers live 68 years with a standard deviation of 9 years,

A) What is the point estimate for the difference of the population means?

B) Find a 95% C.I. for the difference of mean lifetime of non-smokers and smokers.

Given

Population x(non-smokers) n_x=50 , $\bar{x}$ = 76, S_x = 8, $\sigma^2_{\bar{x}} = S^2_x / n_x$, = $8^2/50$ =1.28 years

Population y (smokers) n_y=65 , $\bar{y}$ = 68, S_y = 9, $\sigma^2_{\bar{y}} = S^2_y / n_y$,

7.5.3 Confidence interval for a single proportion

Notation: P (or π) = proportion of "successes" in a population (parameter)

Q = 1-P = proportion of "failures" in a population

p = proportion of successes in a sample

q = 1-p proportion of "failures" in a sample

σ_p= Standard deviation of the sampling distribution of proportions

= Standard error of proportions

n = size of the sample

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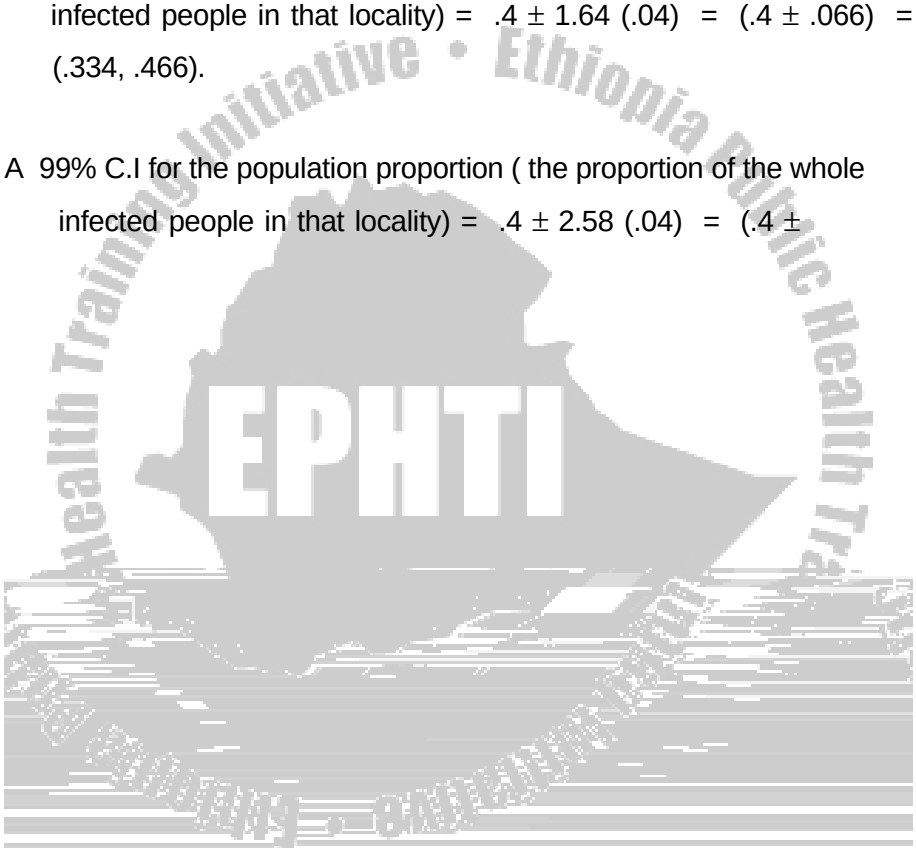
infected people in that locality) = $.4 \pm 1.96$ (.04) = ($.4 \pm .078$) = (.322, .478).

b) A 90% C.I for the population proportion (the proportion of the whole infected people in that locality) = $.4 \pm 1.64$ (.04) = ($.4 \pm .066$) = (.334, .466).

A 99% C.I for the population proportion (the proportion of the whole infected people in that locality) = $.4 \pm 2.58$ (.04) = ($.4 \pm$



confidence limits for the difference in the proportion of all patients with leukaemia who have remission for 2 years.

Note that $n_x p_x = 100 \times .75 = 75 > 5$

$n_x q_y = 100 \times .25 = 25 > 5$

$n_y p_y = 100 \times .60 = 60 > 5$

$n_y q_y = 100 \times .40 = 40 > 5$

$p_x = .75$, $q_x = .25$, $n_x = 100$, $\sigma^2_{Px} = p_x q_x / n_x = .75 \times .25 / 100 = .001875$

$p_y = .60$, $q_x = .40$, $n_y = 100$, $\sigma^2_{Py} = p_y q_y / n_y = .60 \times .40 / 100 = .0024$

Hence, $\sigma^2_{(Px-Py)} = \sqrt{(\sigma^2_{Px} + \sigma^2_{Px})} = \sqrt{(p_x q_x / n_x) + (p_y q_y / n_y)} = \sqrt{.001875+.0024} = .065$

At a 95% Confidence level, Z = ± 1.96 and the difference of the two independent random samples is (.75 - .60) = .15 . Therefore, a 95 % C. I. for the difference in the proportion with 2-year remission is (.15 ± 1.96 (.065)) = (.15 ± .13) = (.02 to .28).

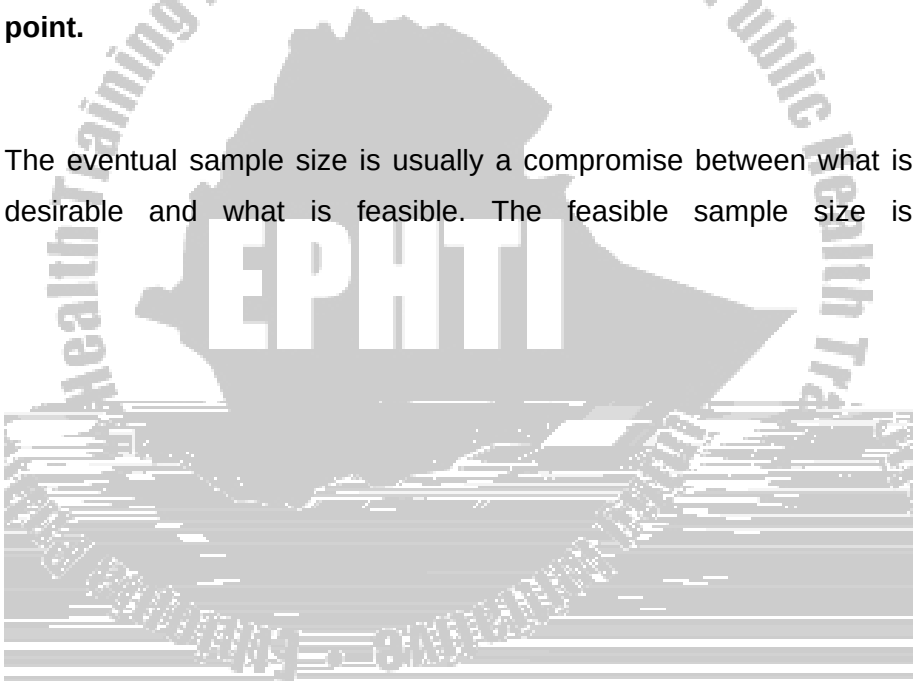
7.6 Sample Size Estimation in cross – sectional studies

In planning any investigation we must decide how many people need to be studied in order to answer the study objectives. If the study is

too small we may fail to detect important effects, or may estimate effects too imprecisely. If the study is too large then we will waste resources.

In general, it is much better to increase the accuracy of data collection (by improving the training of data collectors and data collection tools) than to increase the sample size **after a certain point.**

The eventual sample size is usually a compromise between what is desirable and what is feasible. The feasible sample size is



- the minimum sample size required, for a very large population (N≥10,000)

is:

$$n = Z^2 p(1-p) / w^2$$

Show how the above formula is obtained.

A 95% C.I. for P = p ± 1.96 se , if we want our confidence interval to have a maximum width of ± w,

1.96 se = w

1.96 $\sqrt{}$p(1-p)/n = w

1.96^2 p(1-p)/n = w^2 , Hence, n = 1.96^2 p(1-p)/w^2

Example 1

a) p = 0.26 , w = 0.03 , Z = 1.96 (i.e., for a 95% C.I.)

n = $(1.96)^2$ (.26 × .74) / $(.03)^2$ = 821.25 ≈ 822

Thus , the study should include at least 822 subjects.

b) If the above sample is to be taken from a relatively small population (say N = 3000) , the required minimum sample will be obtained from the

above estimate by making some adjustment .

821.25 / (1+ (821.25/3000)) = 644.7 ≈ 645 subjects

7.6.2 Estimating a mean

The same approach is used but with SE = $\sigma / \sqrt{n}$

The required (minimum) sample size for a very large population is given by:

$$n = Z^2 \sigma^2 / w^2$$

Eg. A health officer wishes to estimate mean haemoglobin level in a defined community. From preliminary contact he thinks this mean is about 150 mg/l with a standard deviation of 32 m/l. If he is willing to tolerate a sampling error of up to 5 mg/l in his estimate, how many subjects should be included in his study? (α =5%, two sided)

- If the population size is assumed to be very large, the required sample size would be:

$n = (1.96)^2 (32)^2 / (5)^2 = 157.4 \approx 158$ persons

- If the population size is , say, 2000 ,
  The required sample size would be 146 persons.

NB: σ^2 can be estimated from previous similar studies or could be obtained by conducting a small pilot study.

7.6.3 Comparison of two Proportions (sample size in each region)

$n = (p_1q_1 + p_2q_2)\ (f(\alpha,\beta))\ /\ ((p_1 - p_2)$

α = type I error (level of significance)

β = type II error (1-β = power of the study)

power = the probability of getting a significant result

f (α,β) =10.5, when the power = 90% and the level of significance = 5%

Eg. The proportion of nurses leaving the health service is compared between two regions. In one region 30% of nurses is estimated to leave the service within 3 years of graduation. In other region it is probably 15%.

Solution

The required sample to show, with a 90% likelihood (power) , that the percentage of nurses is different in these two regions would be: (assume a confidence level of 95%)

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7.7 Exercises

1. Of 45 patients treated by a 1 hour hypnosis session to kick the smoking habit, 36 stopped smoking, at least for the moment. Find a 95% C.I. for the proportion of all smokers who quit after choosing this type of treatment. (the patients were selected randomly).

   A 95% confidence interval for the population proportion (i.e., proportion of all smokers who quit smoking after choosing hypnosis) is (0.68 to 0.92).

2. A hospital administrator wishes to know what proportion of discharged patients are unhappy with the care received during hospitalization. If 95% Confidence interval is desired to estimate the proportion within 5%, how large a sample should be drawn?

$n = Z^2\, p(1-p)/w^2 = (1.96)^2(.5\times.5)/(.05)^2 = 384.2 \approx 385$ patients

NB If you don't have any information about P, take it as 50% and get the maximum value of PQ which is 1/4 (25%).

CHAPTER EIGHT

HYPOTHESIS TESTING

8.1 Learning objectives

At the end of this chapter the student will be able to:

1. Understand the concepts of null and alternative hypothesis
2. Explain the meaning and application of statistical significance
3. Differentiate between type I and type II errors
4. Describe the different types of statistical tests used when samples are large and small
5. Explain the meaning and application of P – values
6. Understand the concepts of degrees of freedom

8.2 Introduction

Definition :A statistical hypothesis is an assumption or a statement which may or may not be true concerning one or more populations.

Eg. 1) The mean height of the Gondar College of Medical Sciences (GCMS) students is 1.63m.

2) There is no difference between the distribution of Pf and Pv malaria in Ethiopia (are distributed in equal proportions.)

In general, hypothesis testing in statistics involves the following steps:

1. Choose the hypothesis that is to be questioned.
2. Choose an alternative hypothesis which is accepted if the original hypothesis is rejected.
3. Choose a rule for making a decision about when to reject the original hypothesis and when to fail to reject it.
4. Choose a random sample from the appropriate population and compute appropriate statistics: that is, mean, variance and so on.
5. Make the decision.

8.3 The null and alternative hypotheses

The main hypothesis which we wish to test is called the null hypothesis, since acceptance of it commonly implies “no effect” or “ no difference.” It is denoted by the symbol H_O.

H_O is always a statement about a parameter (mean, proportion, etc. of a population). It is not about a sample, nor are sample statistics used in formulating the null hypothesis. H_O is an equality ($\mu = 14$) rather than an inequality ($\mu \geq 14$ or $\mu < 14$).

Examples 1) H_O : $\mu = 1.63$ m (from the previous example).

2) At present only 60% of patients with leukaemia survive more than 6 years. A doctor develops a new drug. Of 40 patients, chosen at random, on whom the new drug is tested, 26 are alive after 6 years. Is the new

Choosing the Alternative Hypothesis (H_A)

The notation H_A (or H1) is used for the hypothesis that will be accepted if H_O is rejected. H_A must also be formulated before a sample is tested, so it, like the null hypothesis (H_O), does not depend on sample values. If the mean height of the GCMS students (H_O : μ = 1.63 m) is questioned, then the alternative hypothesis (H_A) is set $\mu \neq 163$ m. Other alternatives are also:

$H_A : \mu > 1.63$ m.

$H_A : \mu < 1.63$ m.

Possible choices of H_A

If H_O is	**then H_A is**
μ = A (single mean)	$\mu \neq A$ or $\mu < A$ or $\mu > A$
P = B (single proportion)	$P \neq B$ or $P < B$ or $P > B$
$\mu_x - \mu_y = C$ (difference of means)	$\mu_x - \mu_y \neq C$ or $\mu_x - \mu_y < C$ or $\mu_x - \mu_y > C$
$P_x - P_y = D$(difference of proportions)	$P_x - P_y \neq D$ or $P_x - P_y < D$ or $P_x - P_y > D$

Where, A, B, C and D are constants.

Consider the previous example (patients with leukaemia)

$$H_O : P = .60$$
$$H_A : P > .60$$

The doctor is trying to reach a decision on whether to make further tests on the new drug. If the proportion of patients who live at least 6 years is not increased under the new treatment or is increased only by an amount due to sampling fluctuation, he will look for another drug. But if the proportion who are aided is significantly larger (that is, if he is able to conclude that the population proportion is greater than .60) - then he will continue his tests.

Exercises

State H_O and H_A for each of the following

1) Is the average height of the GCMS students 1.63 m or is it more?
2) Is the average height of the GCMS students 1.63 m or is it less?
3) Is the average height of the GCMS students 1.63 m or is it something different?
4) There is a belief that 10% of the smokers develop lung cancer in country x.
5) Are men and women infected with malaria in equal proportions, or is a higher proportion of men get malaria in Ethiopia?

8.4 Level of significance

A method for making a decision must be agreed upon. If H_O is rejected, then H_A is accepted. How is a "significant" difference defined? A null hypothesis is either true or false, and it is either rejected or not rejected. No error is made if it is true and we fail to reject it, or if it is false and rejected. An error is made, however, if it is true but rejected, or if it is false and we fail to reject it.

A random sample of size n is taken and the information from the sample is used to reject or accept (fail to reject) the null hypothesis. It is not always possible to make a correct decision since we are dealing with random samples. Therefore, we must learn to live with probabilities of type I (α) and type II (β) errors.

Definitions:

A Type I error is made when H_O is true but rejected.

A Type II error is made when H_O is false but we fail to reject it .

Notation: α is the probability of a type I error. It is called the **level of significance.**

β is the probability of a type II error.

The following table summarises the relationships between the null hypothesis and the decision taken .

Null hypothesis	Decision	
	Accept H_O (Fail to reject H_O)	Reject HO
H_O true	Correct	Type I error
H_O false	Type II error	Correct

In practice, the level of significance (α) is chosen arbitrarily and the limits for accepting H_O are determined. If a sample statistic is outside those limits, H_O is rejected (and H_A is accepted). The form of H_A will determine the kind of limits to be set up (either one tailed or two tailed tests) .

Consider the situation when H_A includes the symbol " $\neq$ " . That is, H_A: $\mu \neq$ …, $P \neq$ …, $\mu_x - \mu_y \neq$ …., $P_x - P_y \neq$ …. etc (two tailed test)

1. α

The most common values of z are:

α	.10	.05	.01
Z	± 1.64	± 1.96	± 2.58

3. The experiment is carried out and the Z value of the appropriate sample statistic (x, p, x-y, p_x - p_y) is determined. If the computed Z value falls within the limits determined in step 2 above, we fail to reject H_O; if the computed Z value is outside those limits, H_O is rejected (and H_A is accepted). Since they separate the "fail to reject" and " reject" regions, the limits determined in step 2 will be referred to as the critical values of Z.

8.5 Tests of Significance on means and Proportions (large samples)

It is important to remember that a test of significance always refers to a null hypothesis. The concern here is with an unknown population parameter, and the null hypothesis states that it is some particular value.

The test of significance answers the question: Is chance (sampling) variation a likely explanation of the discrepancy between a sample result and the corresponding null hypothesis population value? A "yes"

answer – a discrepancy that is likely to occur by chance variation– indicates the sample result is compatible with the claim that the sampling is from a population in which the null hypothesis prevails. This is the meaning of "not statistically significant." A "no" answer – a discrepancy that is unlikely to occur by chance variation – indicates that the sample result is not compatible with the claim that sampling is from a population in which the null hypothesis prevails. This is the meaning of "statistically significant."

As shown earlier, the level significance selected, be it 5 percent, 1 percent, or otherwise, must be clearly indicated. A statement that the results were "statistically significant" without giving further details is worthless.

P – Values

P – values abound in medical and public health research papers, so it is essential to understand precisely what they mean.

Having set up the null hypothesis, we then evaluate the probability that we could have obtained the observed data (or data that were more extreme) if the null hypothesis were true. This probability is usually called the P – value. If it is small, conventionally less than 0.05, the null hypothesis is rejected as implausible. In other words, an outcome that could occur less than one time in 20 when the null hypothesis is true

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A statistical test of significance on a single mean

One begins with a statement that claims a particular value for the unknown population mean. The statistical inference consists of drawing one of the following two conclusions regarding this statement:

I) Reject the claim about the population mean because there is sufficient evidence to doubt its validity.

II) Do not reject the claim about the population mean, because there is not sufficient evidence to doubt its validity.

The analysis consists of determining the chance of observing a mean as deviant as or more deviant than the sample mean, under the assumption that the sample came from a population whose mean is μ_O. One then compares this chance with the predetermined "sufficiently small" chance by referring to the table of the Z distribution (the standard normal distribution) . The critical ratio (Z statistic) is calculated as: $Z = (\bar{x} - \mu_O) / (\sigma / \sqrt{n})$.

Example: Assume that in a certain district the mean systolic blood pressure of persons aged 20 to 40 is 130 mm Hg with a standard deviation of 10 mm Hg . A random sample of 64 persons aged 20 to 40 from village x of the same district has a mean systolic blood pressure of 132 mm Hg. Does the mean systolic blood pressure of the dwellers

of the village (aged 20 to 40) differ from that of the inhabitants of the district (aged 20 to 40) in general, at a 5% Level of significance?

H

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Example: Among susceptible individuals exposed to a particular infectious agent, 36 percent generally develop clinical disease. Among

proportions is again calculated, but because we are evaluating the probability of the data on the assumption that the null hypothesis is true we calculate a slightly different standard error. If the null hypothesis is true, the two samples come from populations having the same true proportion of individuals with the characteristic of interest, say, P . We do not know P, but both p_1 and p_2 are estimates of P. Our best estimate of P is given by calculating :

$$p = \frac{r1 + r2}{n1 + n2}$$

The standard error of P_1 - P_2 under the null hypothesis is thus calculated on the assumption that the proportion in each group is p , so that we have

$$se(P_1 - P_2) = \sqrt{\qquad 8\ 27423.011\ 0\ 10.98\ 274.0574876\ 281.377\ -26377\ -26}$$

came to understand that the proportion of people having malaria in Ethiopia was 3.8% in 1978 (Eth. Cal). The size of the sample considered was 15000. He also realised that during the year that followed (1979), blood samples were taken from 10,000 randomly selected persons. The result of the 1979 seasonal blood survey showed that 200 persons were positive for malaria. Help the health officer in testing the hypothesis that the malaria situation of 1979 did not show any significant difference from that of 1978 (take the level of significance, α =.01).

$H_O : P_{1978} = P_{1979}$ (or $P_{1978} - P_{1979} = 0$)

$H_A : P_{1978} \neq P_{1979}$ (or $P_{1978} - P_{1979} \neq 0$)

$p_{1978} = .038$, $n_{1978} = 15{,}000$

p_{1979}

Decision: reject H_o (because Z calc > Z tab); in other words, the p-value is less than the level of significance (i.e., $\alpha = .01$)

Therefore, it is concluded that there was a statistically significant difference in the proportion of malaria patients between 1978 and 1979 at a .01 level of significance.

Note that the effect of the continuity correction is always to reduce the magnitude of the numerator of the critical ratio. In this sense, the continuity correction is conservative in that its use, compared with its not being used, will label somewhat fewer observed sample differences as being statistically significant. On the other hand, when dealing with values of $n\pi$ and $n(1-\pi)$ that are well above 5, the continuity correction has a negligible effect. This negligible effect can be easily seen from the above example.

8.6 One tailed tests

In the preceding sections we have been dealing with two tailed (sided) tests.

consider the situation when H_A includes the symbol " > or < ". That is,

$H_A: \mu > \_\_$, $H_A: \mu < \_\_$, $H_A : P > \_\_$, $H_A : P < \_\_$, $H_A : \mu_x - \mu_y > \_\_\_$, $H_A : \mu_x - \mu_y < \_\_\_$, etc. (One tailed test).

In the previous sections we learned how to set up a null hypothesis H_O and an alternative hypothesis H_A , and how to reach a decision on

whether or not to reject the null hypothesis at a given level of significance when H_A includes the symbol " $\neq$ ". The decision was reached by following a specific convention (i.e., the area in each tail of the sampling distribution is assumed to be $\alpha/_2$) . This convention determines two values of Z which separate a " fail to reject region" from two rejection regions. Then the Z-value of a statistic is computed for a random sample, and H_O is still accepted if the Z-value falls in the " fail to reject region " previously determined; otherwise, H_O is rejected.

A one tailed test ($H_{A:} \mu >$ __, etc.) is also justified when the investigator can state at the outset that it is entirely inconceivable that the true population mean is below (or above) that of the null hypothesis. To defend this position, there must be solid and convincing supporting evidence. In medical (public health) applications, however, a one tailed test is not commonly encountered.

Example:

Two-tailed test	**one-tailed test**
$H_O: \mu = 100$	$H_O: \mu = 100$
$H_O: \mu \neq 100$	H

dogs for a month; their mean gain in weight per month is 1.15 pounds, with a standard deviation of .30 pound. Does weight gain in 1-year old dogs increase if a special diet supplement is included in their usual diet? (.01 level of significance)

$H_O: \mu = 1.0$

$H_A: \mu > 1.0$ (one tailed test)

Z tab (α =.01) = 2.33 and reject H_O if Z calc > 2.33.

Z calc = (1.15 - 1.0) / (.4/$\sqrt{50}$) = 0.15/0.0566 = 2.65 (This corresponds to a P-value of .004)

Hence, at a .01 level of significance weight gain is increased if a special diet supplement is included in the usual diet of 1-year old dogs.

Example 2: A pharmaceutical company claims that a drug which it manufactures relieves cold symptoms for a period of 10 hours in 90% of those who take it. In a random sample of 400 people with colds who take the drug, 350 find relief for 10 hours. At a .05 level of significance, is the manufacturer's claim correct?

H_O : P = .90

H_A :

The corresponding P-value is .0475

Hence, H_O is rejected: the manufacturer's claim is not upheld.

8.7 comparing the means of small samples

We have seen in the preceding sections how the Standard normal distribution can be used to calculate confidence intervals and to carry out tests of significance for the means and proportions of large samples. In this section we shall see how similar methods may be used when we have small samples, using the t-distribution.

The t-distribution

In the previous sections the standard normal distribution (Z-distribution) was used in estimating both point and interval estimates. It was also used to make both one and two-tailed tests. However, it should be noted that the Z-test is applied when the distribution is normal and the population standard deviation σ is known or when the sample size n is large (**n ≥ 30**) and with unknown σ (by taking S as estimator of σ) .

But, what happens when n<30 and σ is unknown?

We will use a t-distribution which depends on the number of degrees of freedom (df)..The t-distribution is a theoretical probability distribution (i.e, its total area is 100 percent) and is defined by a mathematical function. The distribution is symmetrical, bell-shaped, and similar to the normal but more spread out. For large sample sizes ($n \geq 30$), both t and Z curves are so close together and it does not much matter which you use. As the degrees of freedom decrease, the t-distribution becomes increasingly spread out compared with the normal. The sample standard deviation is used as an estimate of σ (the standard deviation of the population which is unknown) and appears to be a logical substitute. This substitution, however, necessitates an alteration in the underlying theory, an alteration that is especially important when the sample size, n, is small.

Degrees of Freedom

As explained earlier, the t-distribution involves the degrees of freedom (df). It is defined as the number of values which are free to vary after imposing a certain restriction on your data.

Example: If 3 scores have a mean of 10, how many of the scores can be freely chosen?

Solution

The first and the second scores could be chosen freely (i.e., 8 and 12, 9 and 5, 7 & 15, etc.) But the third score is fixed (i.e., 10, 16, 8, etc.) Hence, there are two degrees of freedom.

Exercise: If 5 scores have a mean of 50, how many of the scores can be freely chosen? Find the degrees of freedom.

Table of t-distributions

The table of t-distribution shows values of t for selected areas under the

a) 95% C.I. for the population mean, $\mu = \bar{x} \pm \{t_\alpha (n-1)df \times (S/\sqrt{n})\}$, where,

t tab (with $\alpha = .05$ and $(n-1)df = \pm 2.31$ and $S/\sqrt{9} = 2.89$

Therefore, 95% C.I. for $\mu = 68.7 \pm (2.31 \times 2.89)$

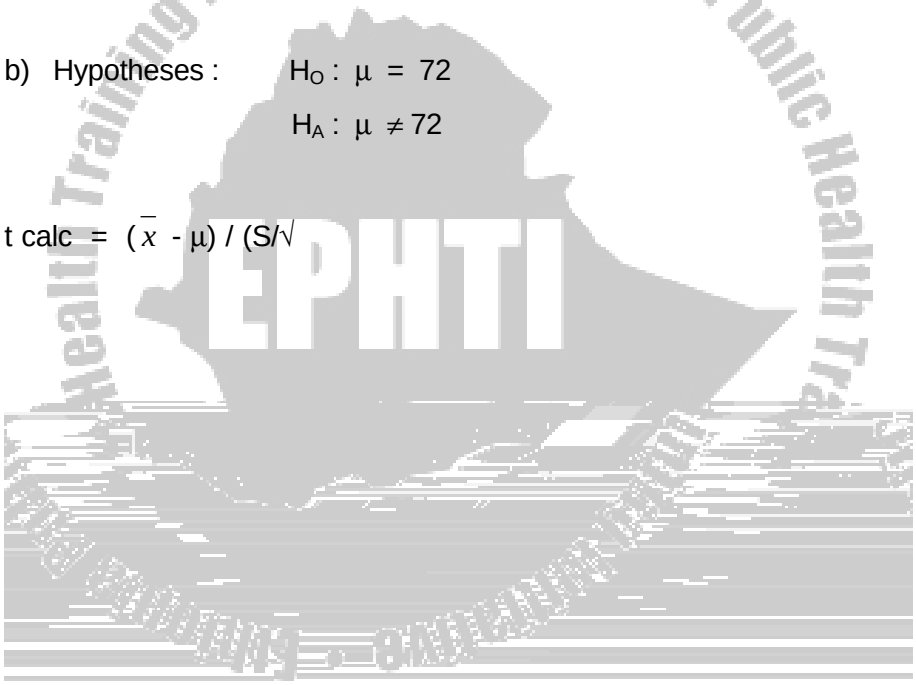
$= 68.7 \pm 6.7$

= (62.0 to 75.4) beats per minute.

b) Hypotheses : $H_O : \mu = 72$

$H_A : \mu \neq 72$

t calc = $(\bar{x} - \mu) / (S/\sqrt{}$



randomly selected male medical students did not show any significant difference from the total mean pulse rate of male medical students at 5% level of significance.

Paired t-test for difference of means

The characteristic feature of paired samples is that each observation in one sample has one and only one mate or matching observation in the other sample. The comparison of means for paired observations is simple and reduces to the methods already discussed. The key to the analysis is that concern is only with the difference for each pair. The null hypothesis states that the population mean difference ($\mu_d = 0$).

Formula for hypothesis tests: $t = (d - \mu_d) / (S_d / \sqrt{n})$ and C.I. for $\mu_d = d \pm t_\alpha (S_d / \sqrt{n})$

Where, d = difference in each pair

$\bar{d}$ = the mean of these differences

S_d = the standard deviation of these differences

$\mu_o d$

Subject	HR before	HR after	difference
1	68	74	+6
2	64	68	+4
3	52	60	+8
4	76	72	- 4
5	78	76	- 2
6	62	68	+6
7	66	72	+6
8	76	76	0
9	78	80	+2
10	60	64	+4
Mean	68	71	+3

A) Does caffeinated coffee have any effect on the heart rate of young men ?

(level of significance = .05)

B) Find the 95% C.I. for the mean of the population differences.

solutions

A) $H_O : \mu_d = 0$

$H_{A:}\ \mu_d \neq 0$

$\bar{d} = 3, \quad \mu_d = 0, \quad S_d = \sqrt{\{\sum(di-d)^2}$ 22 0 2

t calc = ($\bar{d}$ - μ_d) / (S_d / √n) = (3- 0) / (3.92 / √ 10) = 3 / 1.24 = 2.4

(This corresponds to a P-value of less than .05)

t tab (α = .05, df = 9) = 2.26

t calc is > t tab $\Rightarrow$ reject H_O.

Hence, caffeinated coffee changes the heart rate of young men.

B) 95% C.I. for the mean of the population differences = d $\pm$ 2.26 (S_d / √n)

= 3 $\pm$ 2.26(1.24) = 3 $\pm$ 2.8

= (0.2 , 5.8)

Exercise Consider the above data on heart rate. Find the confidence intervals and test the hypothesis when the level of significance takes the values .10, .02 and .01. What do you understand from this?

Two means - unpaired t-test (Independent samples)

The unpaired t-test is one of the most commonly used statistical tests. Unless, specifically stated, when a t-test is discussed, it usually refers

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Mean	**68**	**75**
Variance	**31.11**	**28.67**

A) Test the hypothesis that caffeine has no effect on the pulse rates of young men ($\alpha = .05$).

B) Find the 95% C.I. for the population mean difference.

Before we perform the unpaired t-test we need to know if we have satisfied the necessary assumptions:

1. The groups must be independent. This is ensured since the subjects were randomly assigned.

2. We must have metric (interval or ratio) data.

3. The theoretical distribution of sample means for each group must be normally distributed (we can rely on the central limit theorem to satisfy this).

4. We need assumption of equal variance in the two groups (Homogeneity of variance).

Since the assumptions are met, we can conduct a two-tailed unpaired t-test .

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Hence, caffeinated coffee has an effect on the pulse rates of young men.

B) 95 % C.I. for the population mean difference = (75-68) ± (2.10 x 2.445)

= 7 ± 5.13 = (1.87, 12.13) beats/ minute. That is, there is a 95% certainty that the population mean difference lies between 1.87 and 12.13 beats / minute.

Exercises

For the data given in the above example,

i) Find the 90% and 99% confidence intervals for the population mean difference.
ii) Test the null hypothesis when α takes the values .1 and .01.
iii) What do you understand from your answers.

8.8 Confidence interval or p – value?

The key question in most statistical comparisons is whether an observed difference between two groups of subjects in a sample is large enough to be evidence of a true difference in the population from which the sample was drawn. As shown repeatedly in the previous sections there are two standard methods of answering this question.

A 95% confidence interval gives a plausible range of values that should contain the true population difference. On average, only 1 in 20 of such confidence intervals should fail to capture the true difference. If the 95% confidence interval includes the point of zero difference then, by convention, any difference in the sample cannot be generalized to the population.

A P-value is the probability of getting the observed difference, or more extreme, in the sample purely by chance from a population where the true difference is zero. If the P-value is greater than 0.05 then, by

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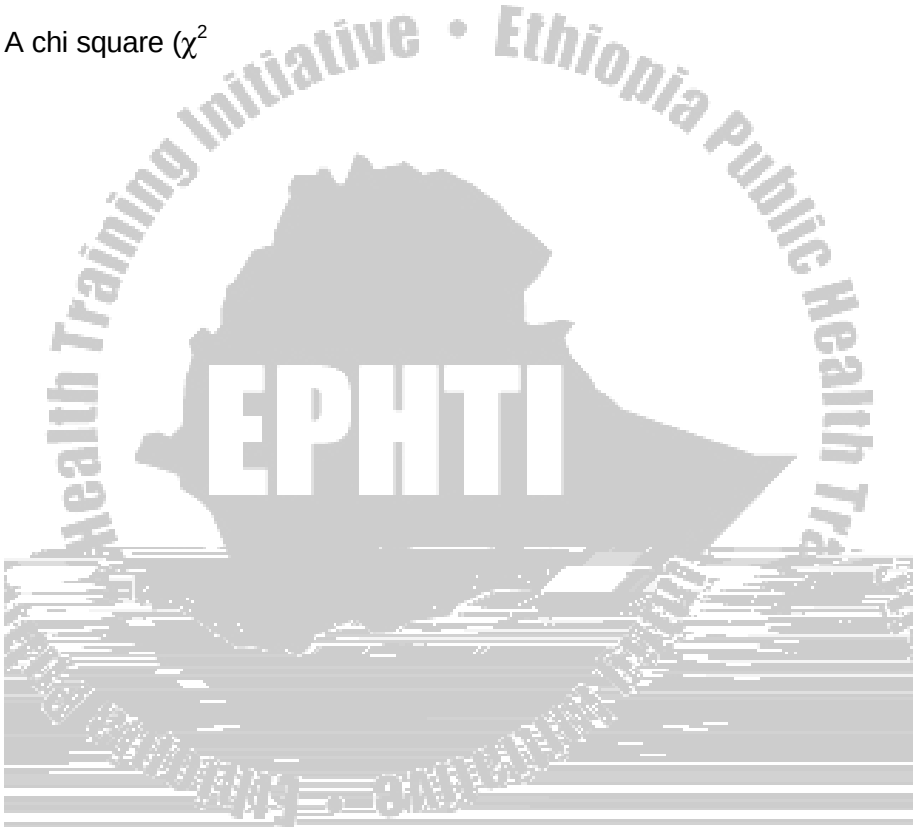
8.9 Test of significance using the chi-square and fisher's exact tests

8.9.1 The Chi – square test

A chi square (χ^2



3. As df increases, the χ^2 curves get more bell shaped and approach the normal curve in appearance (but remember that a chi square curve starts at 0, not at $-\infty$)

If the value of χ^2 is zero, then there is a perfect agreement between the observed and the expected frequencies. The greater the discrepancy between the observed and expected frequencies, the larger will be the value of χ^2.

In order to test the significance of the χ^2, the calculated value of χ^2 is compared with the tabulated value for the given df at a certain level of significance.

Example1: In an experiment with peas one observed 360 round and yellow, 130 round and green, 118 wrinkled and yellow and 32 wrinkled and green. According to the Mendelian theory of heredity the numbers should be in the ratio 9:3:3:1. Is there any evidence of difference from the plants at 5% level of significance?

Solution

Hypothesis: **H_O: Ratio is 9:3:3:1**

H_A: Ratio is not 9:3:3:1

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Observed frequencies

Age of driver

Number of accidents	18 - 25	26 - 40	> 40	total
0	75	115	110	300
1	50	65	35	150
≥ 2	25	20	5	50
Total	150	200	150	500

Expected frequencies

Age of driver

Number of accidents	18 - 25	26 - 40	> 40	total
0	90	120	90	300
1	45	60	45	150
≥ 2	15	20	15	50
Total	150	200	150	500

Calculation of expected frequencies: A total of 150 drivers aged 18-25, and 300/500 = 3/5 of all drivers have had no accidents. If there is no

relation between driver age and number of accidents, we expect that 3/5(150) = 90 drivers aged 18-25 would have no accidents. I.e.,

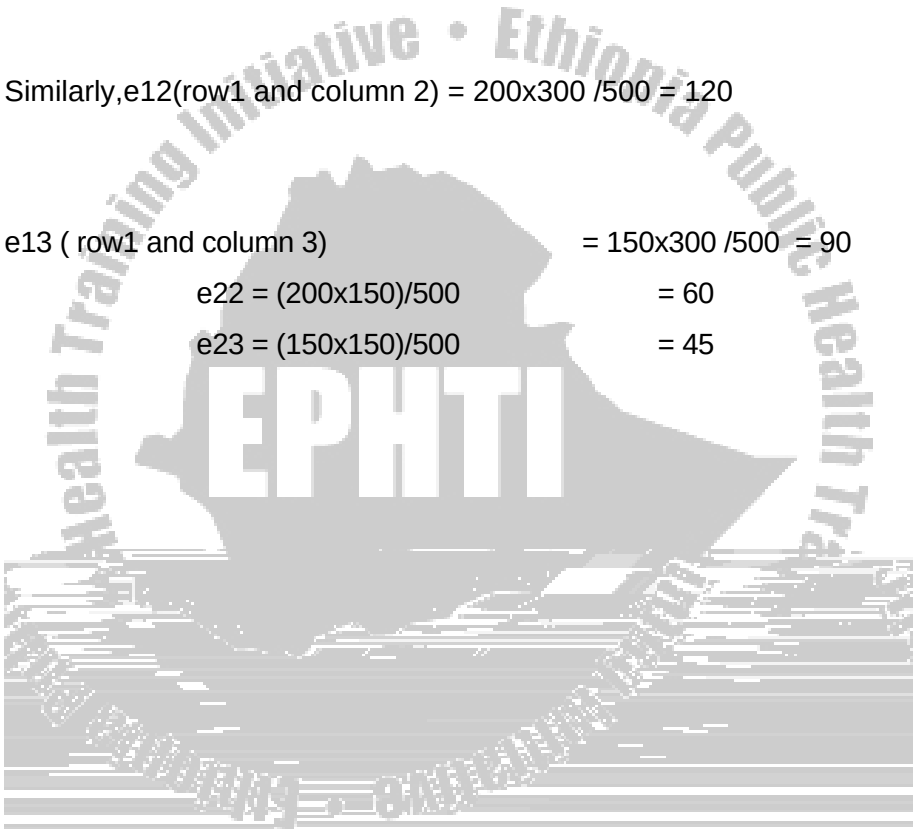
$$e11 = \frac{150 \times 300}{500} = 90$$

Similarly,e12(row1 and column 2) = 200x300 /500 = 120

e13 (row1 and column 3) = 150x300 /500 = 90

e22 = (200x150)/500 = 60

e23 = (150x150)/500 = 45



χ^2 calc = (75 –90)² /90 + (115 – 120)² /120 + (110 – 90)² /90 + … + (5 – 15)² /15

= 1 + 0.208 + 4.444 + 0.556 + 0.417 + 2.222 + 6.667 + 0 + 6.667

= 22. 2 (This corresponds to a P-value of less than .001)

Therefore, there is a relationship between number of accidents and age of the driver.

8.9.2 Fisher's exact test

The chi-square test described earlier is a large sample test. The conventional criterion for the χ^2 test to be valid (proposed by W.G. Cochran and now widely accepted) says that at least 80 percent of the expected frequencies should exceed 5 and all the expected frequencies should exceed 1. Note that this condition applies to the expected frequencies, not the observed frequencies. It is quite acceptable for an observed frequency to be 0, provided the expected frequencies meet the criterion.

If the criterion is not satisfied we can usually combine or delete rows and columns to give bigger expected values. However, this procedure cannot be applied for 2 by 2 tables.

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The exact probability for any given table is now determined from the following formula:

r1! r2! c1! c2! / N! a! b! c! d!

The exclamation mark denotes “factorial” and means successive multiplication by cardinal numbers in descending series, that is 5! means 5x4x3x2x1= 120, By convention 0! = 1.

There is no need to enumerate all the possible tables. The probability of the observed or more extreme tables arising by chance can be found from the simple formula given above.

Pr (observed table) = 8! 4! 6! 6! / 12! 3! 5! 3! 1! = .24

Pr (more extreme table) = 8! 4! 6! 6! / 12! 2! 6! 4! 0! = .03

Consequently, the probability that the difference in mortality between the two treatments is due to chance is **2 x (.24 +.03) = .54**

Hence, the hypothesis that there is no association between treatment and survival cannot be rejected.

NB: If the total probability is small (say less than .05) the data are inconsistent with the null hypothesis and we can conclude that there is evidence that an association exists.

8.10 Exercises

probability that such a difference between sickness rates in the two departments would have arisen by chance?

CHAPTER NINE

CORRELATION AND REGRESSION

9.1 Learning objectives

At the end of this chapter the student will be able to:

1. Explain the meaning and application of linear correlation
2. Differentiate between the product moment correlation and rank correlation
3. Understand the concept of spurious correlation
4. Explain the meaning and application of linear regression
5. Understand the use of scatter diagrams
6. Understand the methods of least squares

9.2 Introduction

In this chapter we shall see the relationships between different variables and closely related techniques of correlation and linear regression for investigating the linear association between two continuous variables. Correlation measures the closeness of the association, while linear regression gives the equation of the straight line that best describes it and enables the prediction of one variable from the other. For example, in the laboratory, how does an animal's

response to a drug change as the dosage of the drug changes? In the clinic, is there a relation between two physiological or biochemical determinations measured in the same patients? In the community, what is the relation between various indices of health and the extent to which health care is available? All these questions concern the relationship between two variables, each measured on the same units of observation, be they animals, patients, or communities. Correlation and regression constitute the statistical techniques for investigating such relationships.

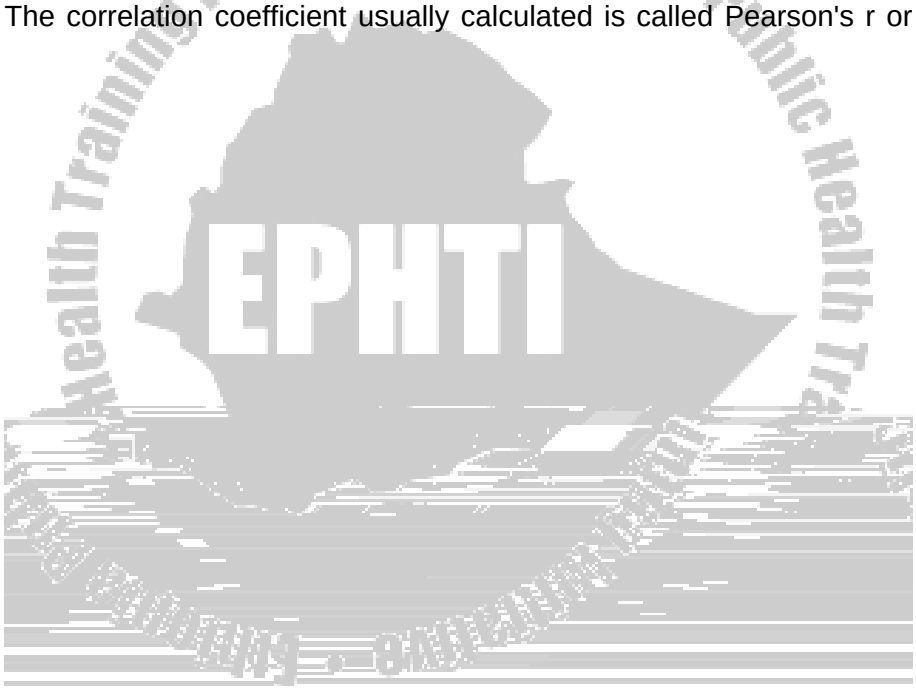
9.3 Correlation Analysis

Correlation is the method of analysis to use when studying the possible association between two continuous variables. If we want to measure the degree of association, this can be done by calculating the correlation coefficient. The standard method (Pearson correlation) leads to a quantity called r which can take any value from -1 to +1. This correlation coefficient r measures the degree of 'straight-line' association between the values of two variables. Thus a value of +1.0 or -1.0 is obtained if all the points in a scatter plot lie on a perfect

correlation of around zero indicates that there is no linear relation between the values of the two variables (i.e. they are uncorrelated).

What are we measuring with r? In essence r is a measure of the scatter of the points around an underlying linear trend: the greater the spread of the points the lower the correlation.

The correlation coefficient usually calculated is called Pearson's r or



Example: Resting metabolic rate (RMR) is related with body weight.

Body Weight (kg)	**RMR (kcal/24 hrs)**
57.6	1325
64.9	1365
59.2	1342
60.0	1316
72.8	1382
77.1	1439
82.0	1536
86.2	1466
91.6	1519
99.8	1639

First we should plot the data using scatter plots. It is conventional to plot the Y- response variable on vertical axis and the independent horizontal axis.

The plot shows that body weight tends to be associated with resting metabolic rate and vice versa. This association is measured by the correlation coefficient, r.

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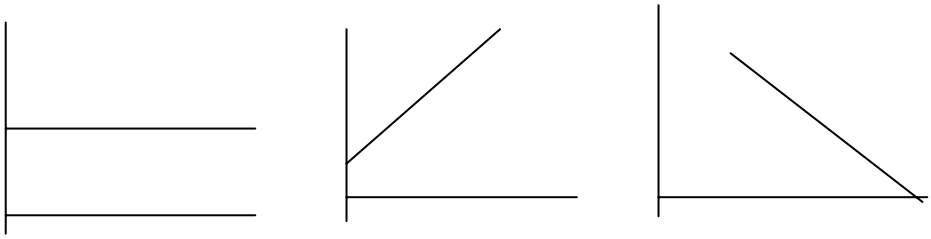


No correlation (r=0) Imperfect +ve correlation (0<r<1) Imperfect –ve correlation (-1<r<0)

Example: The correlation coefficient for the data on body weight and RMR will be:

$$\sum x = 751.20, \sum x^2 = 58,383.7, \sum y = 14,329, \sum y^2 = 20,634,449, \sum xy = 1,089,905.2$$

$$\sum (x-\bar{x})(y-\bar{y}) = \sum xy - (\sum x)(\sum y)/n = 1,089,905.2 - (751.2)(14,329)/10 = 13,510.72$$

$$\sum (x-\bar{x})^2 = \sum x^2 - (\sum x)^2/n = 58,383.7 - (751.2)^2/10 = 1953.56$$

$$\sum (y-\bar{y})^2 = \sum y^2 - (\sum y)^2/n = 20,634,449 - (14,329)^2/10 = 102,424.9$$

$$r = \frac{13,510.72}{\sqrt{(1953.56)(101,424.9)}} = 0.955$$

Hypothesis test

Under the null hypothesis that there is no association in the population (ρ=0) it can be shown that the quantity $r\sqrt{\frac{n-2}{1-r^2}}$ has a t distribution with n-2 degrees of freedom. Then the null hypothesis can be tested by looking this value up in the table of the t distribution.

For the body weight and RMR data:

$$t = r \times \sqrt{\frac{n-2}{1-r^2}} = 0.955 \times \sqrt{\frac{10-2}{1-(0.955)^2}} = 9.11$$

P < 0.001, i.e., the correlation coefficient is highly significantly different from 0.

INTERPRETATION OF CORRELATION

Correlation coefficients lie within the range -1 to +1, with the mid-point of zero indicating no linear association between the two variables. A very small correlation does not necessarily indicate that two variables are not associated, however. To be sure of this we should study a plot of data, because it is possible that the two variables display a non-linear relationship (for example, cyclical or curved). In such cases

association, and so on.

Interpretation of association is often problematic because causation cannot be directly inferred. When looking at variables where there is no background knowledge, inferring a causal link is not justified.

RANK CORRELATION

Occasionally data are not available on the actual measurement of interest, only the relative positions of the members of a group are known. The underlying measurement is continuous - just unobtainable.

For example, James (1985) examined data on dizygotic (DZ) twinning

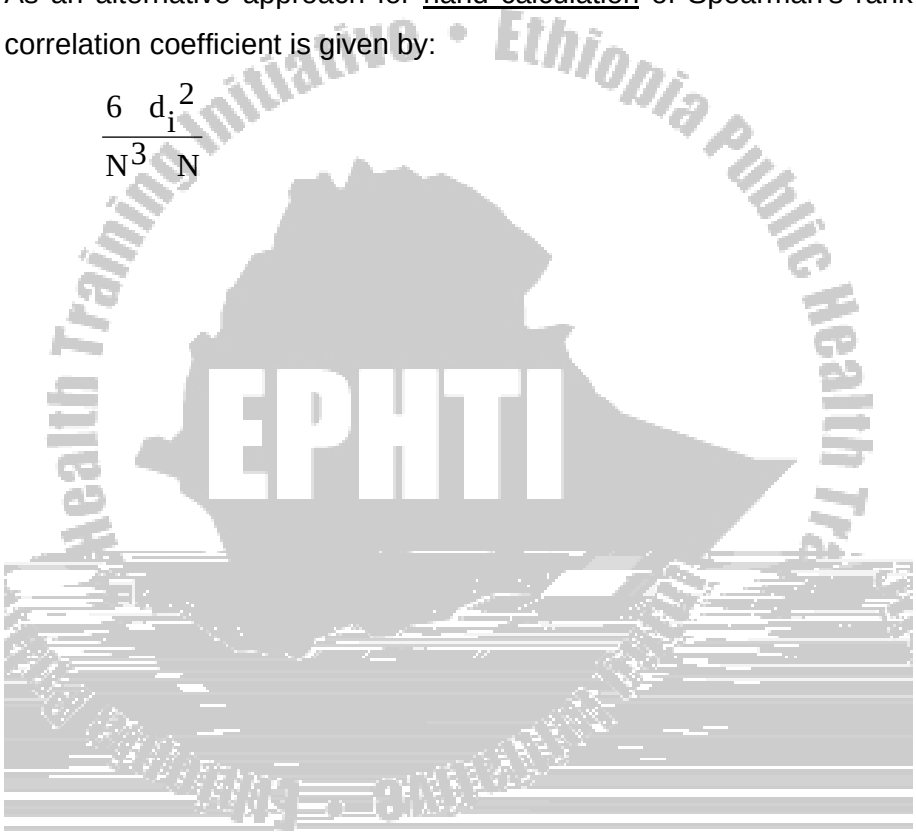
COUNTRY	Latitude	Twining rate	Average milk consumption
Portugal	40 (1.5)	6.5 (2)	3.8
Greece	40(1.5)	8.8(13)	7.7
Spain	41(3)	5.9(1)	8.2
Bulgaria	42(4)	7.0(3)	
Italy	44(5)	8.6(11.5)	6.5
France	47(6.5)	7.1(4)	10.9
Switzerland	47 (6.5)	8.1 (7.5)	
Austria	48 (8)	7.5 (6)	15.9
Belgium	51 (9.5)	7.3 (5)	11.6
FR Germany	51 (9.5)	8.2 (9)	14.1
Holland	52 (11.5)	8.1 (7.5)	18.9
GDR	52 (11.5)	9.1 (16)	
England/Wales	53 (13.5)	8.9 (14.5)	17.1
Ireland	53 (13.5)	11.0 (18)	24.4
Scotland	56 (15.5)	8.9 (14.5)	
Denmark	56 (15.5)	9.6 (17)	16.8
Sweden	60 (17)	8.6 (11.5)	20.9
Norway	61 (18)	8.3 (10)	19.3
Finland	62 (19)	12.1 (19)	30.4

There are two commonly used methods of calculating rank correlation coefficient, one due to Spearman and one due to Kendall. It is generally easier to calculate Spearman's r_s (also called Spearman's

rho). In fact Spearman's rank correlation coefficient is exactly the same as Pearson's correlation coefficient but calculated on the ranks of the observations.

As an alternative approach for hand calculation of Spearman's rank correlation coefficient is given by:

$$\frac{6 \quad d_i^2}{N^3 \quad N}$$



SPURIOUS CORRELATION

The correlation of two variables both of which have been recorded repeatedly over time can be grossly misleading. By such means one may demonstrate relationships between the price of petrol and the divorce rate, consumption of butter and farmers' incomes (a negative relation), and so on. Another example could be the amount of rainfall in Canada and Maize production in Ethiopia (a positive relation).

The same caution applies to studying two variables over time for an individual. Such correlations are often spurious; it is necessary to remove the time trends from such data before correlating them.

9.4 REGRESSION ANALYSIS

The scatter plot of body weight and RMR suggests a linear relationship so we proceed to quantify the relationship between RMR (y) and Body weight (x) by fitting a regression line through the data points. Then the relationship between y and x that is of the following form may be postulated:

$E(Y \mid X) = \alpha + \beta X$ (THAT IS, FOR A GIVEN BODY WEIGHT LEVEL X, THE EXPECTED RESTING METABOLIC RATE $E(Y \mid X)$ IS $\alpha + \beta X$.

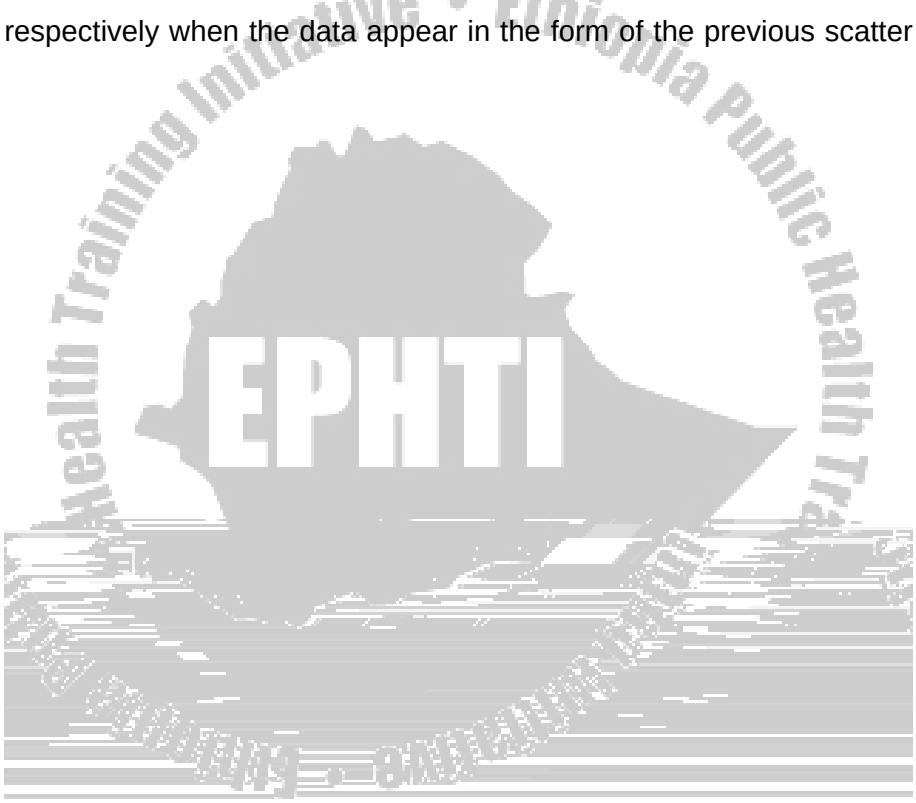
Definition: The line $y = \alpha + \beta x$ is the regression line, where α is the intercept – where the line cuts the y-axis and β is the slope of the line.

The relationship $y = \alpha + \beta x$ is not expected to hold exactly for every individual, an error term e, which represents the variance of RMR among all individuals with a given body weight level x, is introduced

How can β be interpreted? If β is greater than 0, then as x increases, the expected value of $y = \alpha + \beta x$ will increase (e.g. relationship

FITTING REGRESSION LINE—THE METHOD OF LEAST SQUARES

The question remains as to how to fit a regression line (or, equivalently, to obtain estimates of α andβ, denoted by a and b, respectively when the data appear in the form of the previous scatter



derivation, the following least-squares criterion is commonly used.

S = sum of the squared distances of the points from the line

$$= \sum_{i=1}^{n} d_i^2 = \sum_{i=1}^{n} (y_i - a - bx_i)^2$$

Definition: The **least square** line, or **estimated regression line**, is the line y = a + bx that minimizes the sum of squared distances of the sample points from the line given by $S = \sum_{i=1}^{n} d_i^2$. This method of estimating the parameters of the regression line is known as the method of **least squares**.

Based on the least squares estimate, the coefficients of the line y= a + bx are given by:

$$b = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x - \bar{x})^2} = \frac{\sum_{i=1}^{n} x_i y_i - (\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)/n}{\sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2/n}$$ and by substituting the value of b,

$$a = \bar{y} - b\bar{x}$$

Example: The regression line for the data on body weight and RMR will be

$$\sum x = 751.20 \quad \sum x^2 = 58,383.7 \quad \sum y = 14,329 \quad \sum y^2 = 20,634,449 \quad \sum xy = 1,089,905.2$$

$$b = \frac{\sum xy - (\sum x)(\sum y)/n}{\sum x^2 - (\sum x)^2/n} = \frac{10{,}989{,}905.02 - (751.2)(14{,}329)/10}{58{,}383.7 - (751.2)^2/10} = 6.91596$$

$$a = \bar{y} - b\bar{x} = 14{,}329/10 - 6.91596(751.2/10) = 1432.9 - 6.91596(75.12) = 913.3729$$

Thus the regression line is given by $\hat{y} = 913.3729 + 6.91596x$

You recall that we have calculated these results for a random sample of 10 people. Now if we select another sample of 10 people we would get a different estimate for the slope and the intercept. What we are trying to do is to estimate the slope of the "true line". So just like we did when we were estimating the mean or the difference in two means etc. we need to provide a confidence interval for the slope of the line.

This is obtained in the usual wawaETQl wawaET8n8t6ust lik8sing to do is k8sing2.2784 0 5.2

$$\sqrt{-\frac{-}{-}} \qquad \overline{\sqrt{\quad \bar{x})}}$$

where

$$S \quad \sqrt{\frac{(y \quad \bar{y})^2 \quad b^2 \quad (x \quad \bar{x})^2}{n \quad 2}}$$

SIGNIFICANT TEST

Ho: Long-run slope is zero ($\beta=0$)

H_1: Long-run slope is not zero ($\beta\neq0$)

If the null hypothesis is true then the statistic:

$$t = \frac{\text{Observed slope - 0}}{\text{S.E. of obsereved slope}}$$

will follow a t-distribution on n - 2 = 8 degrees of freedom. We lose two degrees of freedom because we have to estimate both the slope and the intercept of the line from the data. A t-distribution on 8 degrees of freedom will have 95% of its area between -2.31 and 2.31. A t-test is used to test whether b differs significantly from a specified value, denoted by β.

$$t = \frac{b - \beta}{s.e.(b)}, df = n - 2$$

For our data set the calculated t-value is:

$$t = \frac{6.92 - 0}{0.754} = 9.18$$

This is very far out in the right-hand tail and is strong evidence against the hypothesis of no relationship. Notice that the output table gives the t-value and a p-value. (Remember that p is the probability of obtaining our result or more extreme given that the null hypothesis is true (true slope = 0)) Again we would then follow this with a confidence interval for the slope.

PREDICTION

DEFINITION: THE PREDICTED, OR EXPECTED, VALUE OF Y FOR A GIVEN VALUE OF X, AS OBTAINED BY THE REGRESSION LINE, IS DENOTED BY $\hat{y}$ =A + BX. THUS THE POINT (X, A + BX) IS ALWAYS ON THE REGRESSION LINE**.**

In some situations it may be useful to use the regression equation to predict the value of y for a particular value of x, say x'. The predicted value is: y' = a + bx' and its standard error is

$$s.e(y') = S\sqrt{1+\frac{1}{n}+\frac{(x'-\bar{x})^2}{\sum(x-\bar{x})^2}}$$

Example: What is the expected RMR if a person has body weight of 65 kg?

IF THE BODY WEIGHT WERE 65 KG, THEN THE BEST PREDICTION OF RMR WOULD BE

$\hat{y}$ =913.3729 + 6.91596(65) = 1362.91 kcal/24 hours.

Standard error of prediction is =

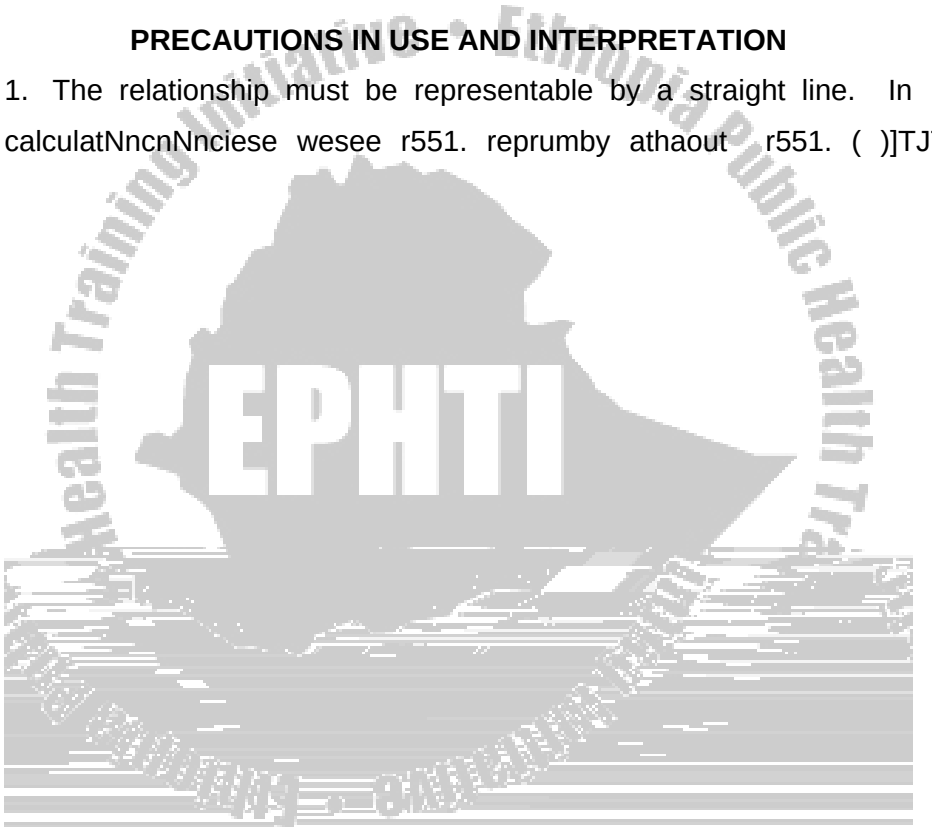
$$33.31 \times \sqrt{1+\frac{1}{10}+\frac{(65-75.12)^2}{1{,}953.556}} = 33.31 \times 1.15 = 38.39$$

Assumptions: There are two assumptions underlying the methods of linear regression: Firstly, for any value of x, y is normally distributed and secondly, the magnitude of the scatter of the points about the line is the same throughout the length of the line. This scatter is

measured by the standard deviation, S, of the points about the line as defined above. A change of scale may be appropriate if either of these assumptions does not hold, or if the relationship seems non-linear.

PRECAUTIONS IN USE AND INTERPRETATION

1. The relationship must be representable by a straight line. In calculatNncnNnciese wesee r551. reprumby athaout r551. ( )]TJT*0 s012 Tc032014 Tw



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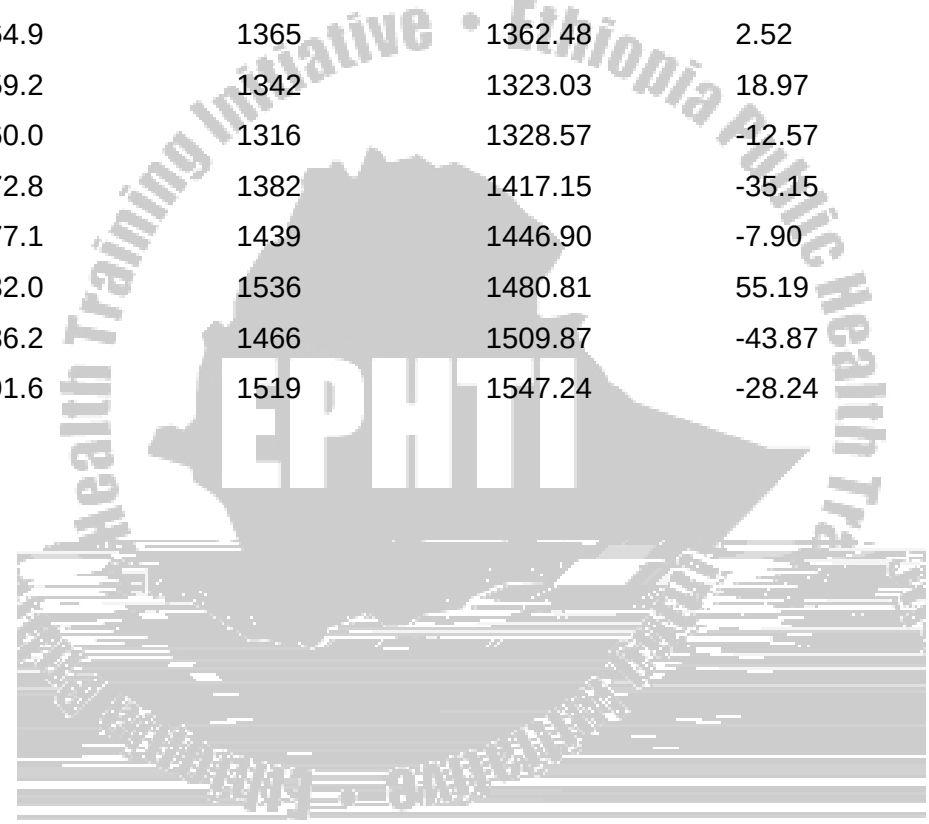
It is usual to eliminate the straight-line effect from the data first and then look for systematic patterns in what is left - the residuals. Subtracting from each observed Y value the corresponding value on the line - the fitted Y value does this:

Residual = Y (observed) – $\hat{Y}$

Fitted (predicted) RMR = 913.37 + 6.92*57.6 = 1311.96

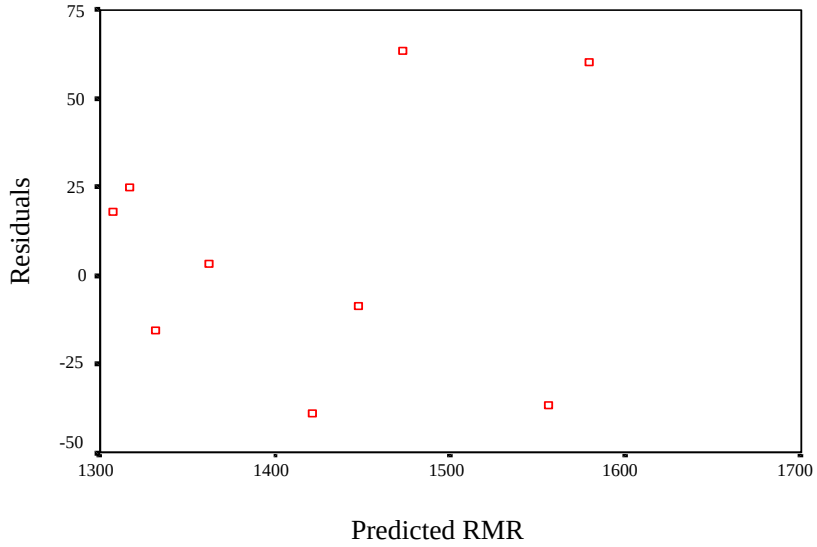
Residual = 1325 - 1311.96 = 13.04

Body weight	Observed RMR	Fitted RMR	Residual
57.6	1325	1311.96	13.04
64.9	1365	1362.48	2.52
59.2	1342	1323.03	18.97
60.0	1316	1328.57	-12.57
72.8	1382	1417.15	-35.15
77.1	1439	1446.90	-7.90
82.0	1536	1480.81	55.19
86.2	1466	1509.87	-43.87
91.6	1519	1547.24	-28.24



Scatterplot of residuals by predicted RMR values

Bi



The residuals are randomly scattered about zero with no discernible trend with predicted values. They neither increase nor decrease systematically as predicted values increase. Neither is there an indication of any non-linear pattern (there is just a hint of a suggestion that the scatter is greater for larger predicted values than for small ones but with so few observations it would be difficult to draw such a conclusion; we should however bear it in mind if more data are to be collected).

APPENDIX : STATISTICAL TABLES

TABLE 4:AREAS IN ONE TAIL OF THE STANDARD NORMAL CURVE

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681

1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.9427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.00130									
3.2	.00069									
3.4	.00034									

3.6	.00016
3.8	.00007
4.0	.00003

Table 5: Percentage points of the t distribution (this table gives the values of t for differing df that cut off specified proportions of the area in one and in two tails of the t distribution)

df	Area in two tails					
	0.2	0.1	0.05	0.02	0.01	0.001
	Area in one tail					
	0.1	0.05	0.025	0.01	0.005	0.0005
1	3.078	6.314	12.706	31.821	63.657	636.619
2	1.886	2.920	4.303	6.965	9.925	31.598
3	1.638	2.353	3.182	4.541	5.841	12.941
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.859
6	1.440	1.943	2.447	3.143	3.707	5.959
7	1.415	1.895	2.365	2.998	3.499	5.405
8	1.397	1.860	2.306	2.896	3.355	5.041
9	1.383	1.833	2.262	2.821	3.250	4.781
10	1.372	1.812	2.228	2.764	3.169	4.587
11	1.363	1.796	2.201	2.718	3.106	4.437
12	1.356	1.782	2.179	2.681	3.055	4.318

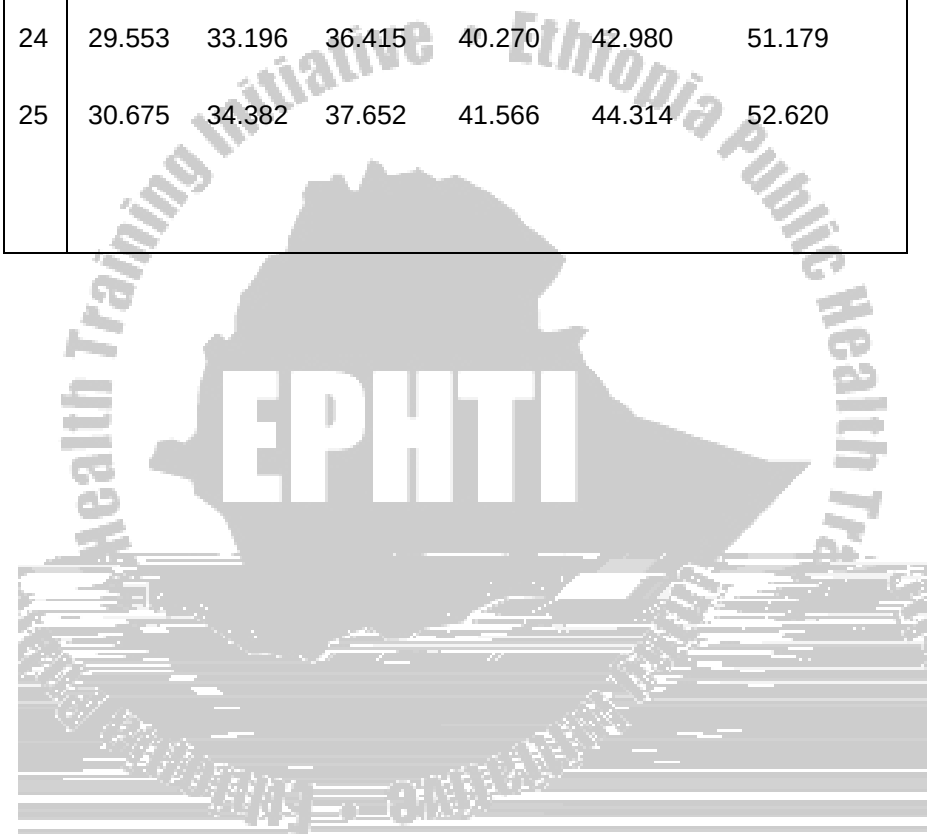
13	1.350	1.771	2.160	2.650	3.012	4.221
14	1.345	1.761	2.145	2.624	2.977	4.140
15	1.341	1.753	2.131	2.602	2.947	4.073
16	1.337	1.746	2.120	2.583	2.921	4.015
17	1.333	1.740	2.110	2.567	2.898	3.965
18	1.330	1.734	2.101	2.552	2.878	3.922
19	1.328	1.729	2.093	2.539	2.861	3.883
20	1.325	1.725	2.086	2.528	2.845	3.850
21	1.323	1.721	2.080	2.518	2.831	3.819
22	1.321	1.717	2.074	2.508	2.819	3.792
23	1.319	1.714	2.069	2.500	2.807	3.767
24	1.318	1.711	2.064	2.492	2.797	3.745
25	1.316	1.708	2.060	2.485	2.787	3.725
26	1.315	1.706	2.056	2.479	2.779	3.707
27	1.314	1.703	2.052	2.473	2.771	3.690
28	1.313	1.701	2.048	2.467	2.763	3.674
29	1.311	1.699	2.045	2.462	2.756	3.659
30	1.310	1.697	2.042	2.457	2.750	3.646
40	1.303	1.684	2.021	2.423	2.704	3.551
60	1.296	1.671	2.000	2.390	2.660	3.460
120	1.289	1.658	1.980	2.358	2.617	3.373
∞	1.280	1.645	1.960	2.326	2.576	3.291

Table 6: Percentage points of the chi-square distribution (this table gives the values of χ^2 for differing df that cut off specified proportions of the upper tail of chi-square the t distribution)

	Area in upper tail					
Df	0.2	0.1	0.05	0.02	0.01	0.001
1	1.642	2.706	3.841	5.412	6.635	10.827
2	3.219	4.605	5.991	7.824	9.210	13.815
3	4.642	6.251	7.815	9.837	11.345	16.268
4	5.989	7.779	9.488	11.668	13.277	18.465
5	7.289	9.236	11.070	13.388	15.086	20.517
6	8.558	10.645	12.592	15.033	16.812	22.457
7	9.803	12.017	14.067	16.622	18.475	24.322

8	11.030	13.362	15.507	18.168	20.090	26.125
9	12.242	14.684	16.919	19.679	21.666	27.877
10	13.442	15.987	18.307	21.161	23.209	29.588
11	14.631	17.275	19.675	22.618	24.725	31.264
12	15.812	18.549	21.026	24.054	26.217	32.909
13	16.985	19.812	22.362	25.472	27.688	34.528
14	18.151	21.064	23.685	26.873	29.141	36.123
15	19.311	22.307	24.996	28.259	30.578	37.697
16	20.465	23.542	26.296	29.633	32.000	39.252
17	21.615	24.769	27.587	30.995	33.409	40.790
18	22.760	25.989	28.869	32.346	34.805	42.312
19	23.900	27.204	30.144	33.687	36.191	43.820
20	25.038	28.412	31.410	35.020	37.566	45.315

21	26.171	29.615	32.671	36.343	38.932	46.797
22	27.301	30.813	33.924	37.659	40.289	48.268
23	28.429	32.007	35.172	38.968	41.638	49.728
24	29.553	33.196	36.415	40.270	42.980	51.179
25	30.675	34.382	37.652	41.566	44.314	52.620



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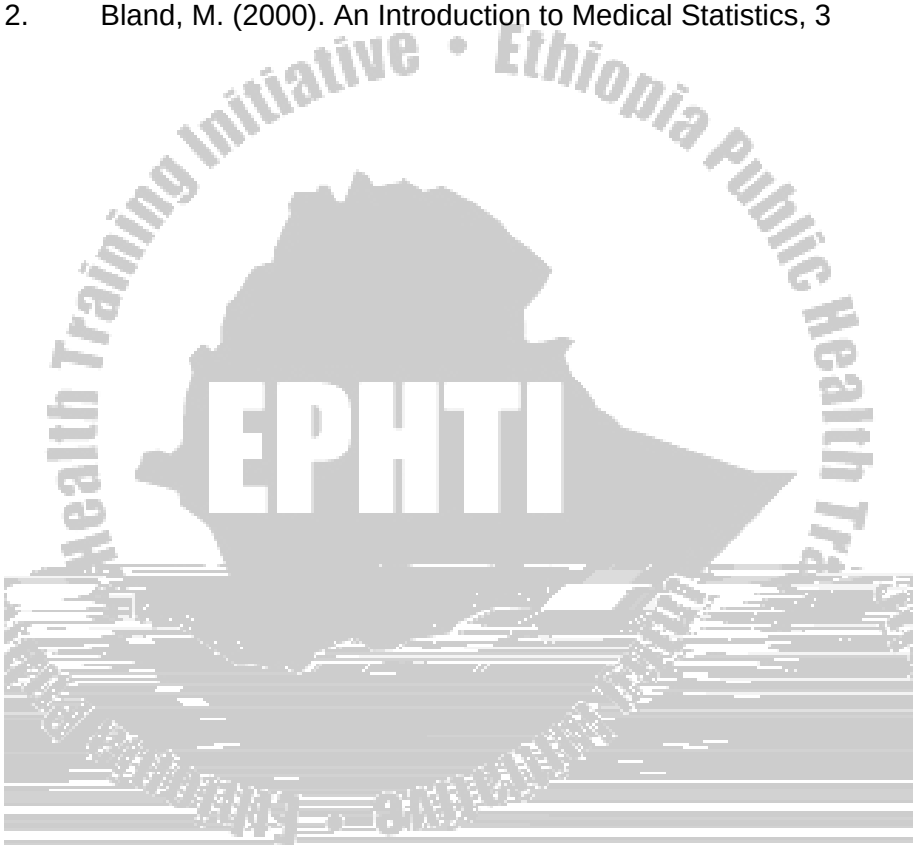

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